

Contents

Preface	v
1 Definitions and basic results	1
1.1 Convex functions	1
1.2 Superquadratic and subquadratic functions	7
1.3 Operator convex functions	8
1.4 Fractional integrals and fractional derivatives	9
2 Some new Hardy-type inequalities with general kernels	15
2.1 Preliminaries	15
2.2 The main results	20
2.3 Remarks and examples	25
3 On an inequality of G. H. Hardy	31
3.1 New inequalities involving fractional integrals and derivatives	43
3.2 Improvements of an inequality of G. H. Hardy	48
4 Some new refined Hardy-type inequalities with kernels	55
4.1 New general refined Hardy-type inequalities with kernels	55
4.2 One-dimensional refined Hardy-Hilbert-type inequalities	71
4.3 Refined Godunova-type inequalities	74
4.4 Refinements of an inequality of G. H. Hardy	77
5 Refinements of Hardy-type inequalities for the case $0 < p \leq q < \infty$	89
5.1 A new class of general Hardy-type inequalities with kernels	89
5.1.1 Further results involving fractional integrals and derivatives	91
5.2 Refined Hardy-type inequalities with kernels	103
5.3 Generalized one-dimensional Hardy's and Pólya-Knopp's inequality	109
5.4 Generalized one-dimensional Hardy-Hilbert's inequality	115
5.5 General Godunova-type inequalities	120
5.6 Generalized G. H. Hardy-type inequality	121
5.6.1 G. H. Hardy-type inequalities for fractional integrals	122
5.6.2 G. H. Hardy-type inequalities for fractional derivatives	127

6	Bounds for Hardy-type differences	129
6.1	The main results with applications	129
6.2	Cauchy means	139
6.3	Further improvements of an inequality of G. H. Hardy	142
6.3.1	Mean value theorems and Cauchy means	148
6.4	n -exponential convexity of Hardy-type functionals	150
6.4.1	The main results	150
6.4.2	Examples	153
7	Hardy-type inequalities with general kernels and measures via superquadratic functions	155
7.1	Preliminaries	155
7.2	The main results with applications	157
7.3	Remarks	163
7.4	Mean value theorems	164
7.5	Exponential convexity	170
7.6	Cauchy means	172
7.7	Inequality of G. H. Hardy and superquadratic functions	174
8	On a new class of refined discrete Hardy-type inequalities	181
8.1	New refined discrete Hardy-type inequalities	182
8.2	Applications. A new refined Carleman's inequality	187
8.3	Exponential convexity and Hardy-type inequalities	195
9	Generalized non-commutative Hardy and Hardy-Hilbert type inequalities	203
9.1	The main results	203
9.2	Remarks end examples	207
10	Boas-type inequalities	213
10.1	A new weighted Boas-type inequality	215
10.2	A new refined weighted Boas-type inequality	222
10.3	A general Boas-type inequality with kernels	225
10.4	Boas-type inequality for superquadratic functions	227
10.4.1	Mean Value Theorems	237
10.4.2	Cauchy Means	239
10.5	Boas-type inequality with constants	240
11	Multidimensional Hardy and Pólya-Knopp-type inequalities	243
11.1	Overview of the Hardy and Pólya-Knopp-type inequalities	243
11.2	Refined Boas inequality with balls in \mathbb{R}^n	247
11.3	Hardy-type inequalities with balls in \mathbb{R}^n	249
11.4	The best constant for Hardy-type inequality	255
11.5	Pólya-Knopp-type inequalities with balls in \mathbb{R}^n	257
11.6	The best constant for the Pólya-Knopp-type inequality	261

12 The Boas functional and its properties	265
12.1 The Boas functional and exponential convexity	265
12.2 Mean value theorems related to the Boas functional	269
12.3 Cauchy-type means generated by the Boas functional	271