

Contents

Preface	v
1 Hilbert-type inequalities with conjugate exponents	1
1.1 Historical overview	2
1.2 A unified treatment of Hilbert-type inequalities with conjugate exponents	5
1.3 Applications to homogeneous kernels	9
1.4 Examples. The best possible constants	14
1.4.1 Integral case	14
1.4.2 Discrete case	18
1.4.3 Some further estimates	25
1.5 Refined Hilbert-type inequalities via the refined Hölder inequality	29
1.6 Multidimensional Hilbert-type inequalities	32
1.6.1 General form	32
1.6.2 Application to homogeneous kernels	35
1.6.3 Examples	38
1.6.4 Inequalities with product-type homogeneous kernels and Schur polynomials	45
2 Hilbert-type inequalities with non-conjugate exponents	53
2.1 General form	55
2.2 The case of a homogeneous kernel	59
2.3 Godunova-type inequalities	64
2.4 Multidimensional case	66
2.5 Examples with hypergeometric functions	73
2.5.1 Hilbert-type inequalities and Gaussian hypergeometric function	74
2.5.2 Hilbert-type inequalities and generalized hypergeometric functions ${}_mF_n$	76
2.6 Hilbert-type inequalities and related operators	84
2.6.1 Hilbert-type inequalities involving a homogeneous function of zero-degree	84
2.6.2 On some related Hilbert-type operators	88

2.6.3	On some related Hardy-type operators	90
2.6.4	Applications	92
3	Hilbert-type inequalities with vector variables	95
3.1	Explicit upper bounds for the doubly weighted Hardy-Littlewood-Sobolev inequality	96
3.2	Trilinear version of standard Beta integral	101
3.3	Multiple Hilbert-type inequalities via the Selberg integral formula	105
3.4	The best constants	111
3.5	Some related inequalities with norms	117
4	Applying the Euler-Maclaurin summation formula	125
4.1	Inequalities for kernels of class C^2	127
4.1.1	Auxiliary results	127
4.1.2	Refined discrete Hilbert-type inequalities	131
4.2	Inequalities for kernels of class C^4	135
4.2.1	Auxiliary results	135
4.2.2	Refined discrete Hilbert-type inequalities	137
4.3	Some particular refinements	142
5	Applying the Hermite-Hadamard inequality	149
5.1	Basic theorem and some remarks	150
5.2	A unified approach to inequalities with a homogeneous kernel	153
5.3	Examples with homogeneous kernels	155
5.4	A non-conjugate example	160
6	Hilbert-type inequalities and the Laplace transform	163
6.1	The case of conjugate parameters	164
6.2	The case of non-conjugate parameters	168
7	A class of Hilbert-Pachpatte-type inequalities	173
7.1	Integral case	174
7.2	Discrete case	179
7.3	Non-conjugate exponents	181
8	General Hardy-type inequalities with non-conjugate exponents	185
8.1	General inequalities of the Hardy-type	187
8.2	General inequalities with dual Hardy-type kernel	191
8.3	Some special Hardy-type kernels and weight functions	193
8.4	Further analysis of parameters	199
8.5	Uniform bounds of constant factors	202
8.6	Applications	205

9 Hilbert-type inequalities in the weighted Orlicz spaces	209
9.1 Weighted Orlicz spaces and a two-dimensional Hilbert-type inequality . .	210
9.2 Multidimensional Hilbert-type inequality	211
9.3 A version of Hardy-Hilbert-type inequality	216
9.4 Some examples in the weighted Lebesgue spaces	218
 10 Some particular inequalities	 221
 Bibliography	 237