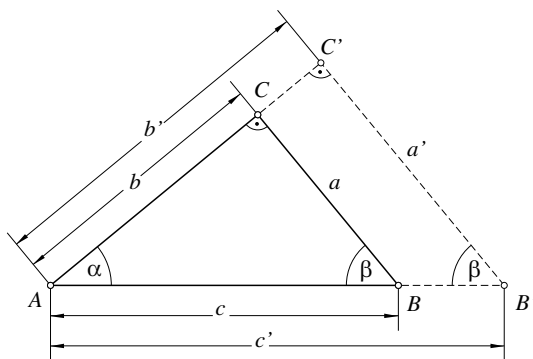


A.

Trigonometrija pravokutnog trokuta

A.1. Definicije trigonometrijskih funkcija šiljastog kuta



Sl. A.1.



Sinus šiljastog kuta u pravokutnom trokutu je omjer duljina kuta nasuprotne katete i hipotenuze. Oznaka: $\sin \alpha$.

Kosinus šiljastog kuta u pravokutnom trokutu je omjer duljina kuta priležeće katete i hipotenuze. Oznaka: $\cos \alpha$.

- A.6.** Odredi vrijednosti trigonometrijskih funkcija kutova α i β pravokutnog trokuta kome je zadano:
- a) $a = 7.3$, $b = 8.1$; b) $a = 2.1$, $b = 6.8$.
- A.7.** Konstruiraj pravokutni trokut ABC ako je:
- a) $\sin \alpha = \frac{3}{5}$, $c = 6$; b) $\cos \alpha = \frac{1}{3}$, $c = 6$;
c) $\operatorname{tg} \alpha = \frac{3}{4}$, $c = 5$; d) $\operatorname{ctg} \alpha = \frac{1}{2}$, $c = 5$.
- A.8.** Konstruiraj pravokutni trokut ABC ako je:
- a) $\sin \beta = \frac{2}{5}$, $b = 4$; b) $\cos \beta = \frac{2}{3}$, $b = 3$;
c) $\operatorname{tg} \beta = 2$, $a = 3$; d) $\operatorname{ctg} \beta = \frac{3}{2}$, $b = 4$.
- A.9.** Konstruiraj pravokutni trokut ABC ako je:
- a) $\operatorname{tg} \alpha = \frac{3}{7}$, $v_c = 2$; b) $\operatorname{ctg} \beta = \frac{4}{5}$, $v_c = \frac{7}{2}$.
- A.10.** Konstruiraj kut φ ako je:
- a) $\sin \varphi = \frac{5}{7}$; b) $\sin \varphi = \frac{\sqrt{3}}{2}$; c) $\cos \varphi = \frac{3}{5}$; d) $\cos \varphi = \frac{\sqrt{2}}{2}$;
e) $\operatorname{tg} \varphi = \frac{2}{3}$; f) $\operatorname{tg} \varphi = 2$; g) $\operatorname{ctg} \varphi = 3$; h) $\operatorname{ctg} \varphi = \sqrt{2}$.
- A.11.** Duljina hipotenuze pravokutnog trokuta je 12, a $\sin \alpha = \frac{3}{4}$. Odredi duljine kateta ovog trokuta.
- A.12.** Duljina jedne katete pravokutnog trokuta je 24, a sinus šiljastog kuta uz tu katetu jednak je $\frac{5}{13}$. Odredi duljinu druge katete i hipotenuze.
- A.13.** Duljina jedne katete pravokutnog trokuta je 8, a kosinus kuta nasuprot toj kateti iznosi $\frac{3}{5}$. Odredi duljinu druge katete i hipotenuze.
- A.14.** Duljina jedne katete pravokutnog trokuta je 42, a tangens kuta nasuprot te katete iznosi $\frac{21}{20}$. Izračunaj duljinu druge katete i hipotenuze.
- A.15.** Duljina hipotenuze pravokutnog trokuta je 34, a tangens jednog šiljastog kuta iznosi $\frac{8}{15}$. Kolika je površina tog trokuta?
- A.16.** Koliki je opseg pravokutnog trokuta kojemu je duljina hipotenuze 82, a kotangens jednog šiljastog jednak $\frac{9}{40}$?
- A.17.** U kojim granicama se kreću vrijednosti funkcija sinus i kosinus šiljastog kuta α u pravokutnom trokutu?

A.18. Za koje $x \in \mathbf{R}$ postoji:

a) $\sin \alpha = \frac{10x}{x^2 + 25}$;

b) $\cos \alpha = \frac{6x}{9 + x^2}$;

c) $\sin \alpha = \frac{2x}{x^2 + 1}$;

d) $\cos \alpha = \frac{1}{2 - x}$?

A.2. Računanje vrijednosti trigonometrijskih funkcija na računalu

A.19. Popuni sljedeću tablicu:

α	$\sin \alpha$	$\cos \alpha$	$\operatorname{tg} \alpha$	$\operatorname{ctg} \alpha$
26°				
39°				
51°				
83°				
$21^\circ 20'$				
$34^\circ 31'$				
$49^\circ 23'$				
$70^\circ 05'$				
$14^\circ 01' 29''$				
$21^\circ 19' 35''$				
$62^\circ 35' 29''$				
$79^\circ 18' 14''$				

A.20. Popuni tablicu:

	α		α
$\sin \alpha = 0.01394$		$\cos \alpha = 0.12345$	
$\sin \alpha = 0.35496$		$\cos \alpha = 0.39011$	
$\sin \alpha = 0.69831$		$\cos \alpha = 0.51348$	
$\sin \alpha = 0.91345$		$\cos \alpha = 0.89390$	
$\operatorname{tg} \alpha = 0.37564$		$\operatorname{ctg} \alpha = 0.11223$	
$\operatorname{tg} \alpha = 1.29389$		$\operatorname{ctg} \alpha = 1.45789$	
$\operatorname{tg} \alpha = 7.41593$		$\operatorname{ctg} \alpha = 6.29354$	
$\operatorname{tg} \alpha = 18.24153$		$\operatorname{ctg} \alpha = 38.21356$	

A.21. Promatrajući tablice iz prethodna dva zadatka što možeš zaključiti o rastu (padu) vrijednosti trigonometrijskih funkcija šiljastog kuta α kad kut raste od 0° prema 90° ?

A.22. Odredi kutove pravokutnog trokuta ako je zadano:

- a)** $a = 6, b = 11$; **b)** $a = 5, c = 13$; **c)** $b = 4, c = 9$;
d) $a = 8, b = 9$; **e)** $a = 2.8, c = 9.3$; **f)** $b = 3.4, c = \frac{28}{3}$.

A.23. Ako je $\sin \alpha = (\operatorname{tg} 21^\circ 21' 21'' + \cos 59^\circ 59' 59'')^2$, koliki je kut α , ($0 < \alpha < 90^\circ$)?

A.24. Ako je $\operatorname{tg} \alpha = (\sin 83^\circ 32' 23'' - \cos 80^\circ 40' 20'')^2$, koliki je kut α , ($0 < \alpha < 90^\circ$)?

A.3. Vrijednost trigonometrijskih funkcija nekih kutova

Vrijednosti trigonometrijskih funkcija sljedećih kutova **treba poznavati "napamet"**:



	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	*
ctg	*	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

* nije definirana.

A.25. Dokaži istinitost gornje tablice.

A.26. Izračunaj:

- a)** $\sin 30^\circ + \cos 30^\circ - \operatorname{tg} 30^\circ - \operatorname{ctg} 30^\circ$;
b) $2 \sin 45^\circ + \operatorname{tg} 45^\circ - 3 \operatorname{ctg} 45^\circ$;
c) $\cos 45^\circ - \operatorname{tg} 45^\circ + 2 \operatorname{ctg} 45^\circ$; **d)** $\sin 60^\circ + 2 \operatorname{tg} 60^\circ - \operatorname{ctg} 60^\circ$.

A.27. Izračunaj:

- a)** $\cos 45^\circ + \operatorname{tg} 60^\circ + \sin 45^\circ - \cos 30^\circ + \cos 60^\circ + \operatorname{ctg} 45^\circ$;
b) $5 \operatorname{tg} 45^\circ + \operatorname{ctg} 30^\circ + 3 \operatorname{tg} 30^\circ - 2 \sin 60^\circ - 2 \cos 30^\circ + 4 \sin 30^\circ$.

A.28. Izračunaj:

- a) $\sin^2 30^\circ + \cos^2 45^\circ + \operatorname{tg}^2 60^\circ$; b) $2 \sin^2 60^\circ + \cos^2 30^\circ + \operatorname{ctg}^2 30^\circ$;
 c) $4 \sin 30^\circ \cos 30^\circ \operatorname{tg} 60^\circ$; d) $3 \operatorname{tg}^2 30^\circ - 4 \sin 45^\circ \cos 60^\circ$.

A.29. Izračunaj:

- a) $(\sin 60^\circ + \sin 45^\circ + \cos 30^\circ) - (\sin 30^\circ \operatorname{tg} 60^\circ - \cos 30^\circ \operatorname{ctg} 60^\circ)$;
 b) $(\operatorname{tg} 45^\circ + \cos 60^\circ \sin 45^\circ + \operatorname{ctg} 60^\circ) - (\sin 60^\circ \cos 45^\circ \operatorname{tg} 30^\circ + \operatorname{ctg}^3 45^\circ)$.

A.30. Izračunaj:

- a) $\frac{2 \sin 30^\circ - 1}{2 \sin 30^\circ + 1}$; b) $\frac{2 \cos 45^\circ + 1}{2 \cos 45^\circ - 1}$;
 c) $\frac{\sin 60^\circ - \sin 30^\circ}{\cos 60^\circ + \cos 30^\circ}$; d) $\frac{\operatorname{tg} 60^\circ - \operatorname{tg} 30^\circ}{\operatorname{ctg} 60^\circ + \operatorname{ctg} 30^\circ}$.

A.31. Izračunaj:

- a) $\frac{5 \operatorname{ctg} 45^\circ - (\sin 60^\circ)^2}{\cos 30^\circ}$; b) $\frac{6 \sin 45^\circ}{\cos 60^\circ + \sin 30^\circ}$.

A.32. Izračunaj:

- a) $\frac{\sin^2 30^\circ + \sin^2 45^\circ}{\cos^2 30^\circ - \cos^2 45^\circ}$; b) $\frac{\operatorname{tg}^2 30^\circ + \operatorname{tg}^2 45^\circ}{\operatorname{ctg}^2 30^\circ - \operatorname{ctg}^2 45^\circ}$;
 c) $\frac{1 - 4 \sin^2 30^\circ}{1 + 4 \cos^2 30^\circ}$; d) $\frac{3 \operatorname{tg}^2 30^\circ + \operatorname{ctg}^2 45^\circ}{\sin^2 60^\circ - \cos^2 45^\circ}$.

A.33. Izračunaj:

- a) $\frac{\sin^2 30^\circ + \sin^2 60^\circ}{\cos^2 30^\circ + \cos^2 60^\circ}$; b) $\frac{\operatorname{tg}^2 30^\circ \operatorname{tg}^2 60^\circ}{\operatorname{ctg}^2 30^\circ \operatorname{ctg}^2 60^\circ}$.

A.34. Pojednostavi:

$$\frac{(p \sin 90^\circ)^2 - (q \operatorname{tg} 45^\circ)^2}{2p^2 \sin 30^\circ - 2pq \cos 0^\circ + q^2 \operatorname{ctg} 45^\circ} \cdot \frac{p - q \operatorname{tg} 45^\circ}{p + q \operatorname{ctg} 45^\circ}.$$

A.35. Provjeri sljedeće jednakosti:

- a) $\cos^2 30^\circ - \sin^2 30^\circ = \cos 60^\circ$;
 b) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = \cos 90^\circ$;
 c) $2 \sin 30^\circ \cos 30^\circ = \sin 60^\circ$; d) $2 \sin 45^\circ \cos 45^\circ = \sin 90^\circ$;
 e) $\sin 30^\circ \cos 60^\circ + \sin 60^\circ \cos 30^\circ = \sin 90^\circ$;
 f) $\sin^2 60^\circ - \cos^2 60^\circ = \sin 30^\circ$;
 g) $\operatorname{tg} 30^\circ \operatorname{ctg} 30^\circ = \sin 90^\circ$; h) $\operatorname{tg} 30^\circ \operatorname{tg} 60^\circ = \cos 0^\circ$;
 i) $\operatorname{tg} 60^\circ \operatorname{ctg} 30^\circ = 3(\sin^2 45^\circ + \cos^2 45^\circ)$;
 j) $\operatorname{ctg} 30^\circ \operatorname{ctg} 60^\circ = (\sin 90^\circ + \cos 90^\circ)^2$.

A.36. Provjeri sljedeće jednakosti:

- a) $\frac{\sin 30^\circ - \cos 30^\circ}{\sin 30^\circ + \cos 30^\circ} = \frac{\operatorname{tg} 30^\circ - 1}{\operatorname{tg} 30^\circ + 1}$; b) $\frac{\operatorname{tg} 60^\circ}{\operatorname{tg} 60^\circ + \operatorname{ctg} 60^\circ} = \cos^2 30^\circ$;

- c) $\frac{1-2\sin^2 45^\circ}{2\cos^2 45^\circ-1} = \operatorname{tg} 45^\circ$; d) $\frac{\sin 30^\circ + \operatorname{ctg} 60^\circ}{\operatorname{tg} 30^\circ} = 1 + \cos 30^\circ$;
- e) $\frac{1 - \sin^4 30^\circ - \sin^2 60^\circ}{\cos^4 30^\circ} = 2 \operatorname{ctg}^2 60^\circ$;
- f) $\frac{\sin 45^\circ}{1 + \operatorname{ctg} 45^\circ} + \frac{\cos 45^\circ}{1 + \operatorname{tg} 45^\circ} = \frac{1}{\sin 45^\circ + \cos 45^\circ}$;
- g) $\frac{\cos^3 30^\circ - \sin^3 30^\circ}{(\cos 30^\circ - \sin 30^\circ) \cdot \cos^2 60^\circ} - \frac{1}{\cos 60^\circ} = \frac{1 + \sin 30^\circ (\sin 60^\circ - 1)}{\cos^2 60^\circ}$;
- h) $\frac{1 - (\cos 45^\circ + \sin 45^\circ)^2}{\sin^2 45^\circ} = -2 \operatorname{ctg} 45^\circ$;
- i) $\frac{\cos^3 30^\circ + \cos^3 60^\circ}{1 - \sin 30^\circ \cos 30^\circ} = \cos 30^\circ + \cos 60^\circ$;
- j) $\sqrt{\frac{1 - \cos 30^\circ}{1 + \sin 60^\circ}} + \sqrt{\frac{1 + \cos 30^\circ}{1 - \sin 60^\circ}} = 4$.

A.37. Na primjerima kutova 30° i 45° provjeri sljedeće identitete:

- a) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$; b) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$;
- c) $\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$; d) $1 + \cos 2\alpha = 2 \cos^2 \alpha$;
- e) $1 - \cos 2\alpha = 2 \sin^2 \alpha$; f) $\frac{\cos \alpha}{1 + \sin \alpha} = \frac{1 - \sin \alpha}{\cos \alpha}$;
- g) $1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$; h) $1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$;
- i) $\frac{\operatorname{ctg}^2 \alpha}{1 + \operatorname{ctg}^2 \alpha} = \cos^2 \alpha$; j) $\frac{\operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha} = \sin^2 \alpha$.

A.4. Osnovne relacije među trigonometrijskim funkcijama

A.38. Dokaži i upamti sljedeće relacije:

$$\begin{array}{l} \text{a) } 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}, \\ \text{b) } 1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}.^* \end{array} \quad (4)$$

* Uvijek treba voditi računa o tome da u nazivniku izraza ne smije biti nula.

A.39. Pojednostavni:

- a)** $1 - \sin^2 \alpha$; **b)** $\cos^2 \alpha - 1$;
c) $\sin \alpha + \operatorname{ctg} \alpha \cos \alpha$; **d)** $\sin^3 \alpha + \sin \alpha \cos^2 \alpha$;
e) $\cos \alpha \sin^2 \alpha + \cos^3 \alpha$; **f)** $\sin \alpha - \sin \alpha \cos^2 \alpha$;
g) $\cos \alpha - \sin^2 \alpha \cos \alpha$.

A.40. Pojednostavni:

- a)** $\operatorname{tg} \alpha \cos \alpha$; **b)** $\operatorname{ctg} \alpha \sin \alpha$; **c)** $\frac{\sin \alpha}{\operatorname{tg} \alpha}$;
d) $\frac{\cos \alpha}{\operatorname{ctg} \alpha}$; **e)** $\frac{\operatorname{tg} \alpha}{\sin \alpha}$; **f)** $\frac{\operatorname{ctg} \alpha}{\cos \alpha}$.

A.41. Pojednostavni:

- a)** $\frac{\sin^2 \alpha}{1 - \cos \alpha}$; **b)** $\frac{\cos \alpha}{\frac{1}{\cos \alpha} + \operatorname{tg} \alpha}$; **c)** $\frac{\cos \alpha}{\sin \alpha} \cdot \frac{1}{\operatorname{ctg}^2 \alpha}$.

A.42. Pojednostavni:

- a)** $\sin^3 \alpha \operatorname{ctg} \alpha + \cos^3 \alpha$; **b)** $\sin^3 \alpha + \cos^3 \alpha \operatorname{tg} \alpha$;
c) $\operatorname{tg} \alpha - \operatorname{tg} \alpha \sin^2 \alpha$; **d)** $\operatorname{ctg} \alpha - \operatorname{ctg} \alpha \cos^2 \alpha$.

A.43. Pojednostavni:

- a)** $\frac{\cos \alpha - \sin \alpha}{1 - \operatorname{tg} \alpha}$; **b)** $\frac{\sin \alpha \operatorname{ctg} \alpha}{\cos \alpha}$;
c) $(1 + \operatorname{tg}^2 \alpha) \cos \alpha$; **d)** $(1 + \operatorname{ctg}^2 \alpha) \sin \alpha$;
e) $\operatorname{tg}^2 \alpha - \operatorname{tg}^2 \alpha \sin^2 \alpha$; **f)** $\operatorname{ctg}^2 \alpha - \operatorname{ctg}^2 \alpha \cos^2 \alpha$.

A.44. Pojednostavni:

- a)** $\sqrt{1 + \sin \alpha} \sqrt{1 - \sin \alpha}$; **b)** $\sqrt{1 - \cos \alpha} \sqrt{1 + \cos \alpha}$;
c) $\frac{\sin^2 \alpha - \sin^4 \alpha}{\cos^2 \alpha - \cos^4 \alpha}$; **d)** $(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2$;
e) $(x \cos \alpha - y \sin \alpha)^2 + (x \sin \alpha + y \cos \alpha)^2$.

A.45. Pojednostavni:

- a)** $\frac{1}{\sin^2 \alpha} - 1$; **b)** $\frac{1 - \cos^2 \alpha}{\sin \alpha \cos \alpha}$; **c)** $\frac{1 + \cos^2 \alpha - \sin^2 \alpha}{\sin^2 \alpha}$;
d) $\frac{1}{1 + \sin \alpha} + \frac{1}{1 - \sin \alpha}$; **e)** $\frac{1}{1 + \cos \alpha} + \frac{1}{1 - \cos \alpha}$.

A.46. Pojednostavni:

- a)** $\frac{\sin \alpha}{\cos^2 \alpha} + \frac{1}{1 + \sin \alpha}$; **b)** $\frac{\cos \alpha}{\sin^2 \alpha} + \frac{1}{1 + \cos \alpha}$;
c) $\frac{1}{1 - \sin \alpha} - \frac{\sin \alpha}{\cos^2 \alpha}$; **d)** $\frac{1}{1 - \cos \alpha} - \frac{\cos \alpha}{\sin^2 \alpha}$;

$$\text{e)} \frac{1 - \cos \alpha}{\sin \alpha} - \frac{\sin \alpha}{1 + \cos \alpha};$$

$$\text{g)} \frac{\cos^2 \alpha}{1 - \sin \alpha} - \frac{\cos^2 \alpha}{1 + \sin \alpha};$$

$$\text{i)} \frac{\cos \alpha}{1 - \operatorname{tg} \alpha} - \frac{\sin \alpha}{\operatorname{ctg} \alpha - 1};$$

$$\text{k)} \frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha};$$

$$\text{f)} \frac{\cos \alpha}{1 - \sin \alpha} - \frac{1 + \sin \alpha}{\cos \alpha};$$

$$\text{h)} \frac{\sin^2 \alpha}{1 - \cos \alpha} - \frac{\sin^2 \alpha}{1 + \cos \alpha};$$

$$\text{j)} \operatorname{ctg} \alpha + \frac{\sin \alpha}{1 + \cos \alpha};$$

$$\text{l)} \frac{\sin \alpha}{1 + \operatorname{ctg} \alpha} + \frac{\cos \alpha}{1 + \operatorname{tg} \alpha}.$$

A.47. Pojednostavni:

$$\text{a)} \frac{\sin^3 \alpha + \sin \alpha \cos^2 \alpha}{\cos \alpha};$$

$$\text{c)} \frac{(\sin \alpha + \cos \alpha)^2 - 1}{\cos^2 \alpha};$$

$$\text{e)} \frac{(\sin \alpha + \cos \alpha)^2 - 1}{\sin^2 \alpha};$$

$$\text{g)} \frac{1}{\cos \alpha} - \frac{\cos \alpha}{1 + \sin \alpha};$$

$$\text{i)} \frac{\sin^2 \alpha}{1 + \sin \alpha} + \frac{\sin^2 \alpha}{1 - \sin \alpha};$$

$$\text{k)} \frac{\sin \alpha}{1 + \sin \alpha} - \frac{\sin \alpha}{1 - \sin \alpha};$$

$$\text{m)} \frac{\cos \alpha}{1 - \sin \alpha} - \frac{\cos \alpha}{1 + \sin \alpha};$$

$$\text{b)} \frac{\cos^3 \alpha + \sin^2 \alpha \cos \alpha}{\sin \alpha};$$

$$\text{d)} \frac{1 - (\sin \alpha - \cos \alpha)^2}{\cos^2 \alpha};$$

$$\text{f)} \frac{1 - (\sin \alpha - \cos \alpha)^2}{\sin^2 \alpha};$$

$$\text{h)} \frac{1}{\cos \alpha} - \frac{\cos \alpha}{1 - \sin \alpha};$$

$$\text{j)} \frac{\cos^2 \alpha}{1 + \cos \alpha} + \frac{\cos^2 \alpha}{1 - \cos \alpha};$$

$$\text{l)} \frac{\cos \alpha}{1 - \cos \alpha} - \frac{\cos \alpha}{1 + \cos \alpha};$$

$$\text{n)} \frac{\sin \alpha}{1 - \cos \alpha} - \frac{\sin \alpha}{1 + \cos \alpha}.$$

A.48. Pojednostavni:

$$\text{a)} \frac{(1 + \sin \alpha)^2 + \cos^2 \alpha}{(1 + \sin \alpha)^2 - \cos^2 \alpha};$$

$$\text{c)} \frac{(1 - \sin \alpha)^2 + \cos^2 \alpha}{(1 - \sin \alpha)^2 - \cos^2 \alpha};$$

$$\text{e)} \frac{1 + \sin \alpha - \cos^2 \alpha}{1 + \sin \alpha};$$

$$\text{g)} \frac{1 - \sin \alpha - \cos^2 \alpha}{1 - \sin \alpha};$$

$$\text{b)} \frac{(1 + \cos \alpha)^2 + \sin^2 \alpha}{(1 + \cos \alpha)^2 - \sin^2 \alpha};$$

$$\text{d)} \frac{(1 - \cos \alpha)^2 + \sin^2 \alpha}{(1 - \cos \alpha)^2 - \sin^2 \alpha};$$

$$\text{f)} \frac{1 + \cos \alpha - \sin^2 \alpha}{1 + \cos \alpha};$$

$$\text{h)} \frac{1 - \cos \alpha - \sin^2 \alpha}{1 - \cos \alpha}.$$

A.49. Pojednostavni:

$$\text{a)} (1 + \operatorname{tg} \alpha)^2 - 2 \operatorname{tg} \alpha;$$

$$\text{c)} (1 - \operatorname{tg} \alpha)^2 + 2 \operatorname{tg} \alpha;$$

$$\text{e)} \frac{1 + \operatorname{tg}^2 \alpha}{1 + \operatorname{ctg}^2 \alpha};$$

$$\text{g)} \frac{1 - \operatorname{ctg}^3 \alpha}{1 - \operatorname{ctg} \alpha} + \frac{1 + \operatorname{ctg}^3 \alpha}{1 + \operatorname{ctg} \alpha};$$

$$\text{b)} (1 + \operatorname{ctg} \alpha)^2 - 2 \operatorname{ctg} \alpha;$$

$$\text{d)} (1 - \operatorname{ctg} \alpha)^2 + 2 \operatorname{ctg} \alpha;$$

$$\text{f)} \frac{1 - \operatorname{tg}^3 \alpha}{1 - \operatorname{tg} \alpha} + \frac{1 + \operatorname{tg}^3 \alpha}{1 + \operatorname{tg} \alpha};$$

$$\text{h) } \frac{1}{(1 + \operatorname{tg} \alpha)^2 + (1 - \operatorname{tg} \alpha)^2} + \frac{1}{(1 + \operatorname{ctg} \alpha)^2 + (1 - \operatorname{ctg} \alpha)^2};$$

$$\text{i) } (\operatorname{tg} \alpha + \operatorname{ctg} \alpha) \sin \alpha \cos \alpha.$$

A.50. Pojednostavni:

$$\text{a) } \frac{1}{\cos^2 \alpha} - \frac{3 \sin \alpha - 1}{1 - \sin \alpha} - \frac{2}{1 + \sin \alpha};$$

$$\text{b) } \frac{1}{\sin^4 \alpha} (1 - \cos^4 \alpha) - 2 \operatorname{ctg}^2 \alpha;$$

$$\text{c) } \sin^2 \alpha \operatorname{tg} \alpha + \cos^2 \alpha \operatorname{ctg} \alpha + 2 \sin \alpha \cos \alpha;$$

$$\text{d) } (\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2;$$

$$\text{e) } 2(\sin^6 \alpha + \cos^6 \alpha) - 3(\sin^4 \alpha + \cos^4 \alpha) + 1;$$

$$\text{f) } 2(\sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha + \cos^4 \alpha)^2 - (\sin^8 \alpha + \cos^8 \alpha);$$

$$\text{g) } 2 \operatorname{tg} \alpha - \frac{2 - \sin \alpha}{\cos^2 \alpha} - \frac{\cos \alpha}{1 - \sin \alpha} + \frac{\sin \alpha + \cos \alpha + 2}{1 + \sin \alpha};$$

$$\text{h) } \frac{1 + \sin \alpha}{\sin^2 \alpha} + \frac{1 + 4 \cos \alpha}{\sin \alpha} - \frac{1 + 3 \sin \alpha - \cos \alpha}{1 - \cos \alpha} + \frac{\sin \alpha}{1 + \cos \alpha};$$

$$\text{i) } \frac{1 + \cos \alpha + \sin^2 \alpha}{\cos^2 \alpha} - \frac{2}{\cos \alpha} - \frac{\sin \alpha}{1 - \sin \alpha} - \frac{1 - \cos \alpha}{1 + \sin \alpha};$$

$$\text{j) } \frac{1 + \cos^2 \alpha}{\sin^2 \alpha} - 4 \operatorname{ctg} \alpha - \frac{2 \cos \alpha + \sin \alpha - 1}{1 - \cos \alpha} - \frac{\cos \alpha - \sin \alpha + 2}{1 + \cos \alpha};$$

$$\text{k) } \frac{1 - (\sin^4 \alpha + \cos^4 \alpha)}{\cos^4 \alpha}; \quad \text{l) } \frac{1 - (\sin^4 \alpha + \cos^4 \alpha)}{\sin^4 \alpha};$$

$$\text{m) } \frac{1}{\cos^2 \alpha} - \frac{\sin^3 \alpha + \cos^3 \alpha}{(\sin \alpha + \cos \alpha) \cos^2 \alpha};$$

$$\text{n) } \frac{\sin^3 \alpha - \cos^3 \alpha}{(\sin \alpha - \cos \alpha) \sin^2 \alpha} - \frac{1}{\sin^2 \alpha};$$

$$\text{o) } \frac{\sin^4 \alpha + \cos^4 \alpha - (\sin^6 \alpha + \cos^6 \alpha)}{\cos^4 \alpha};$$

$$\text{p) } \frac{\sin^6 \alpha + \cos^6 - 1}{\cos^4 \alpha};$$

$$\text{r) } \frac{1 + \sin \alpha}{\cos^2 \alpha} - \frac{4 - 2 \sin \alpha}{\cos \alpha} - \frac{2 \sin \alpha - \cos \alpha - 1}{1 - \sin \alpha} + \frac{3 \cos \alpha}{1 + \sin \alpha};$$

$$\text{s) } \frac{1 + \cos^2 \alpha}{\sin^2 \alpha} + \frac{1 + 3 \cos \alpha}{\sin \alpha} - \frac{2 \sin \alpha + 2 \cos \alpha - 1}{1 - \cos \alpha} - \frac{1 - \sin \alpha}{1 + \cos \alpha}.$$

A.51. Pojednostavni:

$$\text{a) } \frac{\sin^2 \alpha - \sin^2 \beta}{\cos^2 \alpha - \cos^2 \beta}; \quad \text{b) } \frac{\sin^2 \alpha - \cos^2 \beta}{\cos^2 \alpha - \sin^2 \beta}; \quad \text{c) } \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta};$$

- d) $\frac{\sin^4 \alpha - \sin^4 \beta}{\sin^2 \alpha - \sin^2 \beta} + \frac{\cos^4 \alpha - \cos^4 \beta}{\cos^2 \alpha - \cos^2 \beta}$;
- e) $\frac{\sin^4 \alpha - \cos^4 \beta}{\sin^2 \alpha - \cos^2 \beta} + \frac{\cos^4 \alpha - \sin^4 \beta}{\cos^2 \alpha - \sin^2 \beta}$;
- f) $\frac{\sin^2 \alpha - \sin^2 \alpha \cos^2 \beta}{\sin^2 \beta - \cos^2 \alpha \sin^2 \beta}$; g) $\frac{\cos^2 \alpha - \cos^2 \alpha \cos^2 \beta}{\sin^2 \beta - \sin^2 \alpha \sin^2 \beta}$;
- h) $\frac{\cos^2 \alpha - \cos^2 \alpha \sin^2 \beta}{\cos^2 \beta - \sin^2 \alpha \cos^2 \beta}$;
- i) $(\cos \alpha \cos \beta + \sin \alpha \sin \beta)^2 + (\sin \alpha \cos \beta - \cos \alpha \sin \beta)^2$;
- j) $(\cos \alpha \cos \beta - \sin \alpha \sin \beta)^2 + (\sin \alpha \cos \beta + \cos \alpha \sin \beta)^2$.

A.52. Dokaži:

- a) $\frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}$; b) $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = \frac{1}{\sin \alpha \cos \alpha}$;
- c) $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = \frac{\operatorname{ctg} \alpha}{\cos^2 \alpha}$; d) $\frac{\operatorname{tg} \alpha + 1}{\operatorname{tg} \alpha - 1} = \frac{1 + \operatorname{ctg} \alpha}{1 - \operatorname{ctg} \alpha}$;
- e) $(1 + \sin \alpha + \cos \alpha)^2 = 2(1 + \sin \alpha)(1 + \cos \alpha)$;
- f) $(\sin \alpha + \cos \alpha - 1)^2 = 2(\sin \alpha - 1)(\cos \alpha - 1)$;
- g) $\operatorname{tg}^2 \alpha - \sin^2 \alpha = \operatorname{tg}^2 \alpha \sin^2 \alpha$;
- h) $\operatorname{ctg}^2 \alpha \cos^2 \alpha = \operatorname{ctg}^2 \alpha - \cos^2 \alpha$;
- i) $\cos^4 \alpha - \sin^4 \alpha = \cos^2 \alpha(1 - \operatorname{tg} \alpha)(1 + \operatorname{tg} \alpha)$;
- j) $\cos^2 \alpha + \sin^2 \alpha \sin^2 \beta + \sin^2 \alpha \cos^2 \beta = 1$;
- k) $\cos^2 \alpha + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha} - \sin^2 \alpha$;
- l) $\operatorname{tg}^2 \alpha - \operatorname{ctg}^2 \alpha = \frac{1}{\cos^2 \alpha} - \frac{1}{\sin^2 \alpha}$;
- m) $\operatorname{tg} \alpha + \operatorname{tg} \beta = \operatorname{tg} \alpha \operatorname{tg} \beta (\operatorname{ctg} \alpha + \operatorname{ctg} \beta)$.

A.53. Dokaži:

- a) $\frac{\sin^4 \alpha - \cos^4 \alpha}{\sin \alpha - \cos \alpha} = \sin \alpha + \cos \alpha$; b) $\frac{\operatorname{tg} \alpha}{1 + \operatorname{tg}^2 \alpha} = \frac{\operatorname{ctg} \alpha}{1 + \operatorname{ctg}^2 \alpha}$;
- c) $\frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta} = \operatorname{tg} \alpha \operatorname{tg} \beta$; d) $\frac{\sin^3 \alpha}{\cos \alpha - \cos^3 \alpha} = \operatorname{tg} \alpha$;
- e) $(1 - \operatorname{tg} \alpha)(1 - \operatorname{ctg} \alpha) + \frac{1}{\sin \alpha \cos \alpha} = 2$;
- f) $\frac{1}{\cos \alpha} - \cos \alpha = \sin \alpha \operatorname{tg} \alpha$;
- g) $\frac{\sin \alpha}{1 + \cos \alpha} + \frac{1 + \cos \alpha}{\sin \alpha} = \frac{2}{\sin \alpha}$;
- h) $\frac{1}{\sin^2 \alpha \cos^2 \alpha} = \operatorname{tg}^2 \alpha + \operatorname{ctg}^2 \alpha + 2$;

$$\text{i) } \frac{\operatorname{ctg} \alpha \cos \alpha}{\operatorname{ctg} \alpha + \cos \alpha} = \frac{\operatorname{ctg} \alpha - \cos \alpha}{\operatorname{ctg} \alpha \cos \alpha}; \quad \text{j) } \frac{\sin \alpha \operatorname{tg} \alpha}{\sin \alpha + \operatorname{tg} \alpha} = \frac{\operatorname{tg} \alpha - \sin \alpha}{\sin \alpha \operatorname{tg} \alpha};$$

$$\text{k) } \frac{\sin^2 x - \cos^2 x}{\operatorname{tg}^2 x - \operatorname{ctg}^2 x} = \sin^2 x \cdot \cos^2 x; \quad \text{l) } \frac{\operatorname{tg} x}{1 - \operatorname{tg}^2 x} \cdot \frac{\operatorname{ctg}^2 x - 1}{\operatorname{ctg} x} = 1.$$

A.54. Dokaži:

$$\text{a) } \sin^3 \alpha + \cos^3 \alpha = (\sin \alpha + \cos \alpha)(1 - \sin \alpha \cos \alpha);$$

$$\text{b) } (\sin \alpha + \cos \alpha)(\operatorname{tg} \alpha + \operatorname{ctg} \alpha) = \frac{1}{\sin \alpha} + \frac{1}{\cos \alpha};$$

$$\text{c) } (2 - \sin^2 \alpha)(1 + 2 \operatorname{tg}^2 \alpha) = (2 + 2 \operatorname{tg}^2 \alpha)(2 - \cos^2 \alpha);$$

$$\text{d) } (2 - \cos^2 \alpha)(1 + 2 \operatorname{ctg}^2 \alpha) = (2 + \operatorname{ctg}^2 \alpha)(2 - \sin^2 \alpha);$$

$$\text{e) } (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \gamma)^2 + (\sin \alpha \cos \beta - \cos \alpha \sin \beta \cos \gamma)^2 + \sin^2 \beta \sin^2 \gamma = 1.$$

A.55. Dokaži da sljedeći izrazi ne ovise o α :

$$\text{a) } \sin^4 \alpha + \cos^4 \alpha + 2 \sin^2 \alpha \cos^2 \alpha;$$

$$\text{b) } \sin^6 \alpha + \cos^6 \alpha + 3 \sin^2 \alpha \cos^2 \alpha;$$

$$\text{c) } \sin^4 \alpha + \cos^4 \alpha + 2 \sin^2 \alpha \cos^4 \alpha + 2 \sin^4 \alpha \cos^2 \alpha;$$

$$\text{d) } 3(\sin^4 \alpha + \cos^4 \alpha) - 2(\sin^6 \alpha + \cos^6 \alpha).$$

A.56. Dokaži da vrijedi:

$$\frac{\sin x + \operatorname{tg} x}{\cos x + \operatorname{ctg} x} > 0, \quad 0 < x < 90^\circ.$$

A.57. Dokaži da vrijedi:

$$\log \sin x = \log \operatorname{tg} x + \log \cos x, \quad 0 < x < 90^\circ.$$

A.58. Ako je $\sin x + \cos x = a$, $\sin x \cos x = b$, koja relacija povezuje a i b ?

A.59. Ako je $\sin \alpha + \cos \alpha = \frac{4}{3}$, $\sin \beta + \cos \beta = \frac{5}{4}$, koliko je $\sin \alpha \cos \alpha + \sin \beta \cos \beta$?

A.60. Ako je $\sin \alpha \cos \alpha = t$, koliko je $\operatorname{tg} \alpha + \operatorname{ctg} \alpha$?

A.61. Ako je $\operatorname{tg} \alpha + \operatorname{ctg} \alpha = t$, koliko je $\operatorname{tg} \alpha - \operatorname{ctg} \alpha$?

A.62. Ako je $\sin x + \cos x = t$, koliko je:

$$\text{a) } \sin x \cos x; \quad \text{b) } \sin^2 x - \cos^2 x;$$

$$\text{c) } \operatorname{tg} x + \operatorname{ctg} x; \quad \text{d) } \sin^3 x + \cos^3 x?$$

A.63. Ako je $\sin x \cdot \cos x = t$, koliko je:

$$\text{a) } \sin x \cos x; \quad \text{b) } \sin^3 x - \cos^3 x;$$

$$\text{c) } \operatorname{tg} x + \operatorname{ctg} x; \quad \text{d) } \frac{\sin^3 x + \cos^3 x}{\sin^4 x - \cos^4 x}?$$

A.64. Koliko je:

- a) $\frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$ ako je $\operatorname{tg} \alpha = t$;
 b) $\frac{\operatorname{tg} \alpha + \operatorname{ctg} \alpha}{\operatorname{tg} \alpha - \operatorname{ctg} \alpha}$ ako je $\cos \alpha = t$?

A.65. Ako je $\alpha + \beta = 90^\circ$, onda je $\frac{\sin \alpha + \cos \beta}{\cos \alpha + \sin \beta} = \operatorname{ctg} \beta$. Dokaži!

A.66. Odredi vrijednost ostalih trigonometrijskih funkcija šiljastog kuta α ako je $\sin \alpha$ jednak:

- a) $\frac{3}{5}$; b) $\frac{5}{13}$; c) $\frac{8}{17}$; d) $\frac{7}{25}$;
 e) 0.8; f) $\frac{6a}{9+a^2}$; g) $\frac{2a}{a^2+1}$; h) $\frac{2ab}{a^2+b^2}$.

A.67. Odredi vrijednost ostalih trigonometrijskih funkcija šiljastog kuta α ako je $\cos \alpha$ jednak:

- a) $\frac{4}{5}$; b) $\frac{12}{13}$; c) $\frac{20}{29}$; d) $\frac{9}{41}$;
 e) 0.28; f) $\frac{10a}{a^2+25}$; g) $\frac{6a}{9+a^2}$; h) $\frac{a^2-b^2}{a^2+b^2}$.

A.68. Odredi vrijednost ostalih trigonometrijskih funkcija šiljastog kuta α ako je $\operatorname{tg} \alpha$ jednak:

- a) $\frac{4}{3}$; b) $\frac{5}{12}$; c) $\frac{7}{24}$; d) $\frac{9}{40}$;
 e) 0.75; f) $\frac{6a}{9-a^2}$; g) $\frac{a^2-1}{2a}$; h) $\frac{2ab}{a^2-b^2}$.

A.69. Odredi vrijednost ostalih trigonometrijskih funkcija šiljastog kuta α ako je $\operatorname{ctg} \alpha$ jednak:

- a) $\frac{4}{3}$; b) $\frac{12}{5}$; c) $\frac{21}{20}$; d) $\frac{8}{15}$;
 e) 2.4; f) $\frac{a^2-9}{6a}$; g) $\frac{10a}{25-a^2}$; h) $\frac{a^2-4}{4a}$.

A.70. Eliminiraj parametar t iz jednadžbi:

- a) $x = 3 \cos t$; b) $x = r \cos t$; c) $x = 5 \cos t$;
 $y = 3 \sin t$; $y = r \sin t$; $y = 3 \sin t$;
 d) $x = a \cos t$; e) $x = 5 + 3 \cos t$; f) $x = p + r \cos t$;
 $y = b \sin t$; $y = 7 + 3 \sin t$; $y = q + r \sin t$;
 g) $\sin t - \cos t = a$; h) $\operatorname{tg}^2 t + \operatorname{ctg}^2 t = a$;
 $\sin^3 t - \cos^3 t = b$; $\operatorname{tg}^4 t + \operatorname{ctg}^4 t = b$.