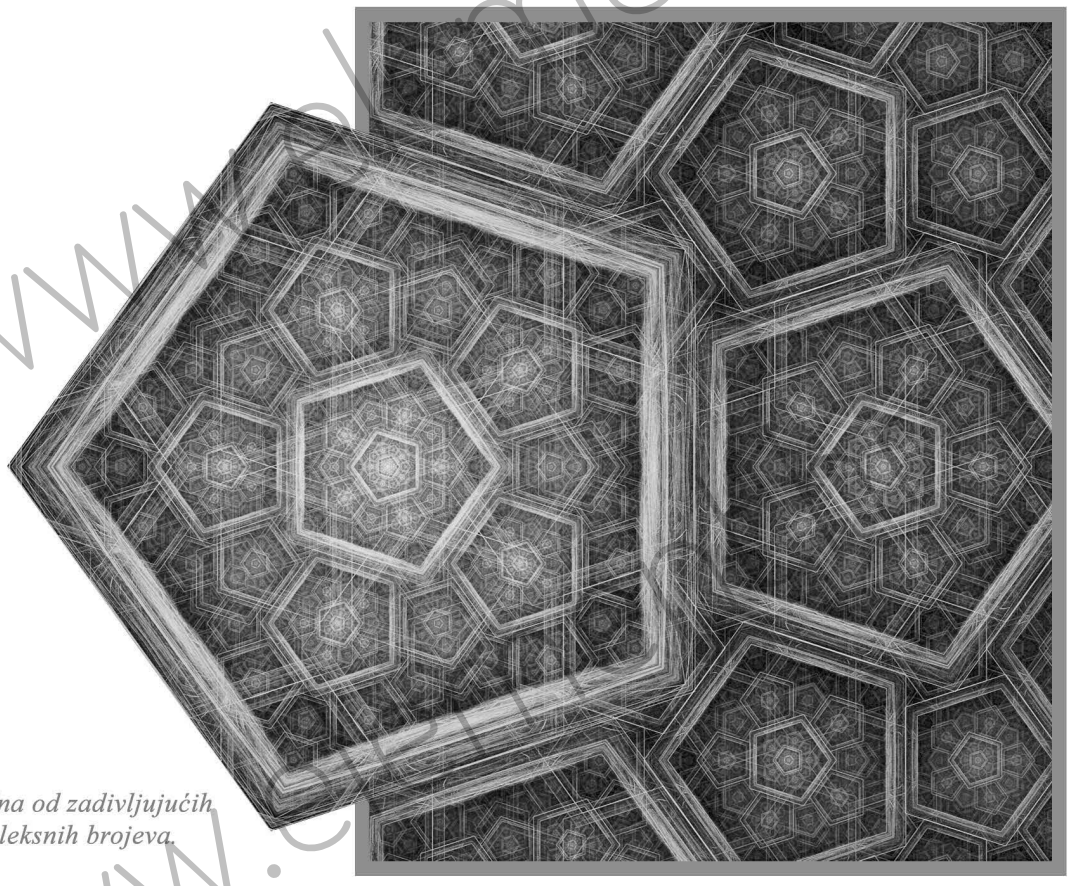
1 Skup kompleksnih brojeva



Fraktali su jedna od zadivljujućih primjena kompleksnih brojeva.

1.1. Kompleksni broj

Zadatak 1. Kvadriraj brojeve:

1) $(-7)^2$; **2)** $\left(-\frac{5}{9}\right)^2$; **3)** $(6i)^2$; **4)** $\left(\frac{3}{7}i\right)^2$;

5) $8i^2$; **6)** $-(9i)^2$; **7)** $(-4i)^2$; **8)** $\frac{5}{8}i^2$;

9) $-\left(\frac{1}{2}i\right)^2$; **10)** $(i\sqrt{5})^2$; **11)** $\left(i\sqrt{\frac{1}{3}}\right)^2$; **12)** $-\left(-\frac{3}{4}i\right)^2$.

Rješenje. **1)** 49; **2)** $\frac{25}{81}$; **3)** -36; **4)** $-\frac{9}{49}$;

5) -8; **6)** 81; **7)** -16; **8)** $-\frac{5}{8}$;

9) $\frac{1}{4}$; **10)** -5; **11)** $-\frac{1}{3}$; **12)** $\frac{9}{16}$.

Zadatak 2. Zapiši brojeve pomoću imaginarne jedinice:

1) $\sqrt{-64}$; **2)** $\sqrt{-\frac{16}{25}}$; **3)** $\sqrt{-\frac{1}{4}}$; **4)** $\sqrt{-7}$;

5) $-\sqrt{-81}$; **6)** $-\sqrt{-\frac{9}{16}}$; **7)** $\sqrt{-0.36}$; **8)** $-\sqrt{-3}$;

9) $-2\sqrt{-144}$; **10)** $3\sqrt{-11}$.

Rješenje.

1) $8i$; **2)** $\frac{4}{5}i$; **3)** $\frac{1}{2}i$; **4)** $i\sqrt{7}$; **5)** $-9i$;

6) $-\frac{3}{4}i$; **7)** $0.6i$; **8)** $-i\sqrt{3}$; **9)** $-24i$; **10)** $3i\sqrt{11}$.

Zadatak 3. Za svaki od brojeva odredi je li realni, imaginarni ili kompleksni broj:

1) $\sqrt{2}$; **2)** $3-2i$; **3)** $-\frac{5}{2}i$; **4)** $i-1$;

5) $0.7+0.5i$; **6)** $\frac{3}{5}$; **7)** 0; **8)** $6i$;

9) $\sqrt{-4}$; **10)** -4; **11)** $1+\sqrt{3}$; **12)** $-i\sqrt{5}$.

Rješenje. Realni brojevi su pod: 1), 6), 7), 10) i 11); imaginarni brojevi su pod: 3), 8), 9) i 12); kompleksni, a da nisu realni i imaginarni, su brojevi pod: 2), 4) i 5).

Zadatak 4. Sljedeću tablicu prepiši u bilježnicu i popuni:

z	$3-7i$	$-1+i\sqrt{5}$	$0.5i$	$\frac{2}{3}-\frac{1}{4}i$	$2-\sqrt{2}$	$-i$	$\pi+i$
$\operatorname{Re} z$							
$\operatorname{Im} z$							

Rješenje.

z	$3-7i$	$-1+i\sqrt{5}$	$0.5i$	$\frac{2}{3}-\frac{1}{4}i$	$2-\sqrt{2}$	$-i$	$\pi+i$
$\operatorname{Re} z$	3	-1	0	$\frac{2}{3}$	$2-2\sqrt{2}$	0	π
$\operatorname{Im} z$	-7	$\sqrt{5}$	0.5	$-\frac{1}{4}$	0	-1	1

Zadatak 5. Odredi realni i imaginarni dio svakog od kompleksnih brojeva:

1) $z=5-2i$; **2)** $z=-5i$; **3)** $z=-1+i$;
4) $z=7.5$; **5)** $z=i$; **6)** $z=0.5-i$.

Rješenje. **1)** $\operatorname{Re} z=5$, $\operatorname{Im} z=-2$; **2)** $\operatorname{Re} z=0$, $\operatorname{Im} z=-5$;
3) $\operatorname{Re} z=-1$, $\operatorname{Im} z=1$; **4)** $\operatorname{Re} z=7.5$, $\operatorname{Im} z=0$;
5) $\operatorname{Re} z=0$, $\operatorname{Im} z=1$; **6)** $\operatorname{Re} z=0.5$, $\operatorname{Im} z=-1$.

Zadatak 6. Odredi realni i imaginarni dio svakog od kompleksnih brojeva:

1) $z=5+2i$; **2)** $z=1-3i$;
3) $z=-\frac{1}{2}i$; **4)** $z=\sqrt{2}$;
5) $z=\frac{2-3i}{3}$; **6)** $z=1-\sqrt{2}+i\sqrt{3}$;
7) $z=0$; **8)** $z=(1-\sqrt{2})i$.

Rješenje. **1)** $\operatorname{Re} z=5$, $\operatorname{Im} z=2$; **2)** $\operatorname{Re} z=1$, $\operatorname{Im} z=-3$;
3) $\operatorname{Re} z=0$, $\operatorname{Im} z=-\frac{1}{2}$; **4)** $\operatorname{Re} z=\sqrt{2}$, $\operatorname{Im} z=0$;
5) $\operatorname{Re} z=\frac{2}{3}$, $\operatorname{Im} z=-1$; **6)** $\operatorname{Re} z=1-\sqrt{2}$, $\operatorname{Im} z=\sqrt{3}$;
7) $\operatorname{Re} z=0$, $\operatorname{Im} z=0$; **8)** $\operatorname{Re} z=0$, $\operatorname{Im} z=1-\sqrt{2}$.

Zadatak 7. Odredi realne brojeve a i b iz jednakosti:

1) $a+4i=-2+bi$;
2) $a+i=1+bi$;
3) $2a-b+3i=a+2b-3i$;
4) $1-(a-b)i=1+(a+b)i$;

5) $a - b + 5i = 1 + (a + b)i$;
6) $-1 + (2a + b)i = 1 - (a - 2b)i$.

Rješenje. **1)** $a = -2$, $b = 4$; **2)** $a = 1$, $b = 1$; **3)** nema rješenja;
4) $a = 0$, b može biti bilo koji realni broj;
5) $a = 3$, $b = 2$. **6)** nema rješenja.

Zadatak 8. Odredi realne brojeve x i y iz jednakosti:

1) $x + (y - 1)i = -1 + 3i$;
2) $2x + y - yi = 1 + i$;
3) $x - y + (x + y)i = 2 + 4i$;
4) $x - 2y + (2x - y)i = 3i$;
5) $2x - 3y + (x - y)i = -1$;
6) $2x - 3y + (x + y)i = x + 2y + (3x + 1)i$.

Rješenje. **1)** Kompleksni brojevi s lijeve i desne strane jednakosti jednaki su ako je $x = -1$ i $y - 1 = 3$, odnosno $x = -1$, $y = 4$;

2) Mora biti $2x + y = 1$ i $-y = 1$, odnosno iz druge jednakosti $y = -1$, pa uvrštavanjem u prvu jednakost imamo $2x - 1 = 1$, $2x = 2$, $x = 1$. Dakle $x = 1$, $y = -1$.

3)
$$\left.\begin{aligned} x - y &= 2 \\ x + y &= 4 \end{aligned}\right\} +$$
$$2x = 6 \implies x = 3$$
$$3 - y = 2 \implies y = 1;$$

4)
$$\begin{aligned} x - 2y &= 0 \\ 2x - y &= 3 \quad / \cdot (-2) \end{aligned}$$
$$\left.\begin{aligned} x - 2y &= 0 \\ -4x + 2y &= -6 \quad / \cdot (-2) \end{aligned}\right\} +$$
$$-3x = -6 \implies x = 2$$
$$2 - 2y = 0 \implies y = 1;$$

5)
$$\begin{aligned} 2x - 3y &= -1 \\ x - y &= 0 \implies x = y \end{aligned}$$
$$2x - 3x = -1 \implies x = 1, \ y = 1$$

6)
$$\begin{aligned} 2x - 3y &= x + 2y \\ x + y &= 3x + 1 \end{aligned}$$
$$\left.\begin{aligned} x - 5y &= 0 \\ -2x + y &= 1 \quad / \cdot 5 \end{aligned}\right\} +$$
$$-9x = 5 \implies x = -\frac{5}{9}$$
$$-2 \cdot \left(-\frac{5}{9}\right) + y = 1 \implies y = -\frac{1}{9}.$$

Zadatak 9. Gdje je greška u računu:

$$\begin{aligned} 1 = \sqrt{1} &= \sqrt{(-1) \cdot (-1)} = \sqrt{-1} \cdot \sqrt{-1} \\ &= i \cdot i = -1? \end{aligned}$$

Rješenje. Svojstvo drugog korijena umnoška, tj. $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ vrijedi samo za pozitivne brojeve a i b. Stoga ne vrijedi jednakost $\sqrt{(-1) \cdot (-1)} = \sqrt{-1} \cdot \sqrt{-1}$.

1.2. Zbrajanje i množenje kompleksnih brojeva

Zadatak 1. Izračunaj $z + w$, $z - w$ i $z \cdot w$ ako je:

1) $z = -\frac{1}{2} + i$, $w = 1 - \frac{1}{3}i$; **2)** $z = -2 + 3i$, $w = 2 + i$;

3) $z = \frac{3}{4} - \frac{2}{3}i$, $w = \frac{3}{4} + \frac{1}{3}i$; **4)** $z = -7 + 5i$, $w = 8 + 11i$;

5) $z = -8 - 3i$, $w = 4 + 5i$; **6)** $z = -6 + 5i$, $w = -5 + 6i$.

Rješenje. **1)** $z + w = -\frac{1}{2} + i + 1 - \frac{1}{3}i = \frac{1}{2} + \frac{2}{3}i$,

$z - w = -\frac{1}{2} + i - \left(1 - \frac{1}{3}i\right) = -\frac{3}{2} + \frac{4}{3}i$,

$z \cdot w = \left(-\frac{1}{2} + i\right)\left(1 - \frac{1}{3}i\right) = -\frac{1}{2} + \frac{1}{6}i + i - \frac{1}{3}i^2 = -\frac{1}{2} + \frac{7}{6}i + \frac{1}{3} = -\frac{1}{6} + \frac{7}{6}i$;

2) $z + w = -2 + 3i + 2 + i = 4i$,

$z - w = -2 + 3i - (2 + i) = -4 + 2i$,

$z \cdot w = (-2 + 3i) \cdot (2 + i) = -4 - 2i + 6i + 3i^2 = -4 + 4i - 3 = -7 + 4i$;

3) $z + w = \frac{3}{4} - \frac{2}{3}i + \frac{3}{4} + \frac{1}{3}i = \frac{3}{2} - \frac{1}{3}i$,

$z - w = \frac{3}{4} - \frac{2}{3}i - \left(\frac{3}{4} + \frac{1}{3}i\right) = -i$,

$z \cdot w = \left(\frac{3}{4} - \frac{2}{3}i\right) \cdot \left(\frac{3}{4} + \frac{1}{3}i\right) = \frac{9}{16} - \frac{1}{4}i - \frac{1}{2}i - \frac{2}{9}i^2 = \frac{9}{16} - \frac{3}{4}i + \frac{2}{9} = \frac{113}{144} - \frac{3}{4}i$.

4) $z + w = -7 + 5i + 8 + 11i = 1 + 16i$,

$z - w = -7 + 5i - 8 - 11i = -15 - 6i$,

$z \cdot w = (-7 + 5i)(8 + 11i) = -56 - 77i + 35i - 55 = -111 - 37i$;

5) $z + w = -8 - 3i + 4 + 5i = -4 + 2i$,

$z - w = -8 - 3i - 4 - 5i = -12 - 8i$,

$z \cdot w = (-8 - 3i)(4 + 5i) = -32 - 40i - 12i + 15 = -17 - 52i$;

6) $z + w = -6 + 5i - 5 + 6i = -11 + 11i$,

$z - w = -6 + 5i + 5 - 6i = -1 - i$,

$z \cdot w = (-6 + 5i)(-5 + 6i) = 30 - 36i - 25i - 30 = -61i$.

Zadatak 2. Izračunaj $z + w$, $z - w$, $z \cdot w$, z^2 i w^2 ako je:

1) $z = 1 - 2i$, $w = 3 - i$;

2) $z = -3 + 5i$, $w = 4 - 7i$;

3) $z = 11 - 5i$, $w = -7 - i$.

Rješenje. **1)** $z + w = 1 - 2i + 3 - i = 4 - 3i$,

$z - w = 1 - 2i - (3 - i) = -2 - i$,

$z \cdot w = (1 - 2i) \cdot (3 - i) = 3 - i - 6i + 2i^2 = 3 - 7i - 2 = 1 - 7i$,

$z^2 = (1-2i)^2 = 1-4i+4i^2 = 1-4i-4 = -3-4i$,
$w^2 = (3-i)^2 = 9-6i+i^2 = 9-6i-1 = 8-6i$;
2) $z+w = -3+5i+4-7i = 1-2i$,
$z-w = -3+5i-(4-7i) = -7+12i$,
$z\cdot w = (-3+5i)(4-7i) = -12+21i+20i-35i^2 = -12+41i+35 = 23+41i$,
$z^2 = (-3+5i)^2 = 9-30i+25i^2 = 9-30i-25 = -16-30i$,
$w^2 = (4-7i)^2 = 16-56i+49i^2 = 16-56i-49 = -33-56i$;
3) $z+w = 11-5i+(-7-i) = 4-6i$,
$z-w = 11-5i-(-7-i) = 18-4i$,
$z\cdot w = (11-5i)(-7-i) = -77-11i+35i+5i^2 = -77+24i-5 = -82+24i$,
$z^2 = (11-5i)^2 = 121-110i+25i^2 = 121-110i-25 = 96-110i$,
$w^2 = (-7-i)^2 = 49+14i+i^2 = 49+14i-1 = 48+14i$.

Zadatak 3. **1)** $(1+i)^2$; **2)** $(1-2i)^2$; **3)** $(2-i)^2$; **4)** $(1+2i)^3$;
5) $(3+2i)^3$; **6)** $(i+2)^3$; **7)** $(1-i)^4$; **8)** $(2+i)^4$.

Rješenje. **1)** $(1+i)^2 = 1+2i+i^2 = 1+2i-1 = 2i$;
2) $(1-2i)^2 = 1-4i+4i^2 = 1-4i-4 = -3-4i$;
3) $(2-i)^2 = 4-4i+i^2 = 4-4i-1 = 3-4i$;
4) $(1+2i)^3 = 1+6i+12i^2+8i^3 = 1+6i-12-8i = -11-2i$;
5) $(3+2i)^3 = 27+54i+36i^2+8i^3 = 27+54i-36-8i = -9+46i$;
6) $(i+2)^3 = i^3+6i^2+12i+8 = -i-6+12i+8 = 2+11i$;
7) $(1-i)^4 = ((1-i)^2)^2 = (1-2i+i^2)^2 = (-2i)^2 = -4$;
8) $(2+i)^4 = ((2+i)^2)^2 = (4+4i-1)^2 = (3+4i)^2 = 9+24i-16 = -7+24i$.

Zadatak 4. **1)** $(1-i\sqrt{2})(\sqrt{2}-i)$; **2)** $(\sqrt{3}-i)(1+i\sqrt{3})$;
3) $(\sqrt{2}-i)(\sqrt{3}+2i)-(\sqrt{3}+i)(\sqrt{2}-i)$.

Rješenje. **1)** $(1-i\sqrt{2})(\sqrt{2}-i) = \sqrt{2}-i-2i+\sqrt{2}i^2 = \sqrt{2}-3i-\sqrt{2} = -3i$;
2) $(\sqrt{3}-i)(1+i\sqrt{3}) = \sqrt{3}+3i-i-\sqrt{3}i^2 = \sqrt{3}+2i+\sqrt{3} = 2\sqrt{3}+2i$;
3) $(\sqrt{2}-i)(\sqrt{3}+2i)-(\sqrt{3}+i)(\sqrt{2}-i) = \sqrt{6}+2\sqrt{2}i-\sqrt{3}i-2i^2-(\sqrt{6}-\sqrt{3}i+\sqrt{2}i-i^2) = \sqrt{6}+2\sqrt{2}i-\sqrt{3}i+2-\sqrt{6}+\sqrt{3}i-\sqrt{2}i-1 = 1+\sqrt{2}i$.

Zadatak 5. **1)** $(1-i)(2-i)(3-i)$; **2)** $(1+i)(1+2i)(1+3i)$;
3) $\left(\frac{1}{2}-i\right)(1+2i)\left(1-\frac{1}{2}i\right)(2+i)$.

Rješenje. **1)** $(1-i)(2-i)(3-i) = (2-3i-1)(3-i) = (1-3i)(3-i) = 3-10i+3 = -10i$;
2) $(1+i)(1+2i)(1+3i) = (1+3i-2)(1+3i) = (3i-1)(3i+1) = 9i^2-1 = -9-1 = -10$;
3) $\left(\frac{1}{2}-i\right)(1+2i)\left(1-\frac{1}{2}i\right)(2+i) = \left(\frac{1}{2}+i-i+2\right)\left(2+i-i+\frac{1}{2}\right) = \frac{5}{2}\cdot\frac{5}{2} = \frac{25}{4}$.

Zadatak 6. Izračunaj vrijednost brojevnog izraza
$z^2 - z\cdot w + w^2$ ako je:

1) $z = 1 - 2i$, $w = 3 + i$;
2) $z = \sqrt{2} - i$, $w = \sqrt{2} + i$.

Rješenje. **1)** $(1-2i)^2-(1-2i)(3+i)+(3+i)^2 = 1-4i-4-3-i+6i-2+9+6i-1 = 7i$;
2) $(\sqrt{2} - i)^2 - (\sqrt{2} - i) \cdot (\sqrt{2} + i) + (\sqrt{2} + i)^2$
$= 2 - 2i\sqrt{2} - 1 - 2 - 1 + 2 + 2i\sqrt{2} - 1 = -1$.

Zadatak 7. Dani su kompleksni brojevi $z = -5 + 8i$ i $w = 7 - 11i$. Odredi realni i imaginarni dio brojeva

1) $z^2 - w^2$; **2)** $(z - w)^2$; **3)** $z^2 + w^2$.

Rješenje. **1)** $z^2 - w^2 = (z - w)(z + w) = (-5 + 8i - 7 + 11i)(-5 + 8i + 7 - 11i)$
$= (-12 + 19i)(2 - 3i) = -24 + 36i + 38i + 57 = 33 + 74i$
$\operatorname{Re}(z^2 - w^2) = 33$, $\operatorname{Im}(z^2 - w^2) = 74$;
2) $(z - w)^2 = (-5 + 8i - 7 + 11i)^2 = (19i - 12)^2 = -361 - 456i + 144$
$= -217 - 456i$
$\operatorname{Re}(z - w)^2 = -217$, $\operatorname{Im}(z - w)^2 = -456$;
3) $z^2 + w^2 = (-5 + 8i)^2 + (7 - 11i)^2 = -64 - 80i + 25 + 49 - 154i - 121$
$= -111 - 234i$
$\operatorname{Re}(z^2 + w^2) = -111$, $\operatorname{Im}(z^2 + w^2) = -234$.

Zadatak 8. Brojevi $z_1 = 3 + 4i$ i $z_2 = 3 - 4i$ rješenja su jednadžbe $z^2 - 6z + 25 = 0$. Provjeri.

Rješenje.

$$\begin{aligned} z_1^2 - 6z_1 + 25 &= 0 \\ (3 + 4i)^2 - 6(3 + 4i) + 25 &= 0 \\ 9 + 24i - 16 - 18 - 24i + 25 &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} z_2^2 - 6z_2 + 25 &= 0 \\ (3 - 4i)^2 - 6(3 - 4i) + 25 &= 0 \\ 9 - 24i - 16 - 18 + 24i + 25 &= 0 \\ 0 &= 0 \end{aligned}$$

Zadatak 9. Brojevi $z_1 = 2 - 3i$ i $z_2 = 2 + 3i$ rješenja su jednadžbe $z^2 - 4z + 13 = 0$. Provjeri.

Rješenje.

$$\begin{aligned} z_1^2 - 4z_1 + 13 &= 0 \\ (2 - 3i)^2 - 4(2 - 3i) + 13 &= 0 \\ 4 - 12i - 9 - 8 + 12i + 13 &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} z_2^2 - 4z_2 + 13 &= 0 \\ (2 + 3i)^2 - 4(2 + 3i) + 13 &= 0 \\ 4 + 12i - 9 - 8 - 12i + 13 &= 0 \\ 0 &= 0 \end{aligned}$$

Zadatak 10. Odredi realne brojeve x i y iz jednakosti:

1) $(1 - i)x + (1 + i)y = i$;
2) $(2 - 3i)x - (1 + 4i)y = 3 + i$;
3) $(x + y)(2 - i) + (x - y)(1 + 3i) = 2 + 3i$.

Rješenje. **1)**

$$\begin{aligned}(1-i)x+(1+i)y&=i\\ x-xi+y+yi&=i\\ x+y&=0\\ -x+y&=1\\ \hline 2y&=1\\ y&=\frac{1}{2}\\ x&=-\frac{1}{2}\end{aligned}$$

2)

$$\begin{aligned}(2-3i)x+(1+4i)y&=3+i\\ 2x-3xi+y+4yi&=3+i\\ 2x+y&=3\\ -3x+4y&=1\\ \hline -3x+4(3-2x)&=1\\ -3x+12-8x&=1\\ -11x&=-11\\ x&=1\\ y&=1\end{aligned}$$

3)

$$\begin{aligned}(x+y)(2-i)+(x-y)(1+3i)&=2+3i\\ 2x-xi+2y-yi+x-y+3xi-3yi&=2+3i\\ 3x+y&=2\\ 2x-4y&=3\\ \hline 2x-4(2-3x)&=3\\ 2x-8+12x&=3\\ 14x&=11\\ x&=\frac{11}{14}\\ y&=-\frac{5}{14}\end{aligned}$$

Zadatak 11. Riješi sustave jednadžbi:

1) $\begin{cases} z+2w=1+i,\\ 3z+iw=2-3i;\end{cases}$ **2)** $\begin{cases} 2z+w=7i,\\ zi+w=-1;\end{cases}$

3) $\begin{cases} (1-i)z-iw=5-4i,\\ (1+i)z-(1-2i)w=8-i.\end{cases}$

Rješenje. **1)** Neka je $z=a+bi$ i $w=c+di$.

$$\begin{aligned}&(a+bi)+2(c+di)=1+i\\ &3(a+bi)+i(c+di)=2-3i\\ \hline &a+bi+2c+2di=1+i\\ &3a+3bi+ci-d=2-3i\\ \hline &a+2c=1\\ &b+2d=1\\ &3a-d=2\\ &3b+c=-3\\ \hline &3(1-2c)-d=2\\ &3(1-2d)+c=-3\\ &3-6c-d=2\\ &3-6d+c=-3\\ \hline &d=1-6c\\ &-6+36c+c=-6\end{aligned}$$

$$c = 0$$
$$d = 1$$
$$b = -1$$
$$a = 1$$
$$z = 1 - i$$
$$w = i$$

2) Neka je $z = a + bi$ i $w = c + di$.

$$2z + w = 7i$$
$$zi + w = -1$$
$$2z - zi = 7i + 1$$
$$2(a + bi) - (a + bi)i = 7i + 1$$
$$2a + 2bi - ai + b = 7i + 1$$
$$2a + b = 1$$
$$2b - a = 7$$
$$5b = 15$$
$$b = 3$$
$$a = -1$$
$$z = -1 + 3i$$
$$2(-1 + 3i) + (c + di) = 7i$$
$$-2 + 6i + c + di = 7i$$
$$d = 1$$
$$c = 2$$
$$w = 2 + i$$

3) Neka je $z = a + bi$ i $w = c + di$.

$$(1 - i)(a + bi) - i(c + di) = 5 - 4i$$
$$(1 + i)(a + bi) - (1 - 2i)(c + di) = 8 - i$$
$$a + bi - ai + b - ci + d = 5 - 4i$$
$$a + bi + ai - b - c - di + 2ci - 2d = 8 - i$$
$$a + b + d = 5$$
$$-a + b - c = -4$$
$$a - b - c - 2d = 8$$
$$a + b + 2c - d = -1$$
$$2b - c + d = 1$$
$$-2c - 2d = 4$$
$$2b + c - d = -5$$
$$c + d = -2$$
$$-2c + 2d = 6$$

$$2d = 1$$
$$d = \frac{1}{2}$$
$$c = -\frac{5}{2}$$
$$b = -1$$
$$a = \frac{11}{2}$$
$$z = \frac{11}{2} - i$$
$$w = -\frac{5}{2} + \frac{1}{2}i$$

Zadatak 12. Izračunaj:

1) i^{77}; **2)** i^{1359}; **3)** i^{2468}
4) i^{325}; **5)** i^{510}; **6)** i^{707}.

Rješenje.
1) $i^{77} = i^{4\cdot 19+1} = i^{4\cdot 19}\cdot i = (i^4)^{19}\cdot i = 1\cdot i = i$;
2) $i^{1359} = i^{4\cdot 339+3} = i^{4\cdot 339}\cdot i^3 = (i^4)^{339}\cdot(-i) = 1\cdot(-i) = -i$;
3) $i^{2468} = i^{4\cdot 617} = (i^4)^{617} = 1$.
4) $i^{325} = i^{4\cdot 81}\cdot i = 1\cdot i = i$;
5) $i^{510} = i^{4\cdot 127}\cdot i^2 = 1\cdot(-1) = -1$;
6) $i^{707} = i^{4\cdot 176}\cdot i^3 = 1\cdot(-i) = -i$.

Zadatak 13. Koliko je:

1) $i + i^3 + i^5 + i^7 + i^9 + \ldots + i^{33}$;
2) $i^2 + i^4 + i^6 + i^8 + i^{10} + \ldots + i^{30}$;
3) $i^{102} - i^{104} + i^{106} - i^{108} + \ldots + i^{122}$;
4) $i^{101} - i^{103} + i^{105} - i^{107} + \ldots + i^{121}$;
5) $1 + i^3 + i^6 + i^9 + i^{12} + i^{15}$;
6) $i^{111} + i^{222} + i^{333} + \ldots + i^{999}$?

Rješenje.
1) $i + i^3 + i^5 + i^7 + i^9 + \ldots + i^{33} = i - i + i^{4+1} + i^{4+3} + i^{2\cdot 4+1} + \ldots + i^{8\cdot 4+1}$
$= i - i + i - i + i - \ldots + i = i$;
2) $i^2 + i^4 + i^6 + i^8 + i^{10} + \ldots + i^{30} = -1 + 1 + i^{4+2} + i^{2\cdot 4} + i^{2\cdot 4+2} + \ldots + i^{7\cdot 4+2}$
$= -1 + 1 - 1 + 1 - 1 + \ldots - 1 = -1$;
3) $\underbrace{i^{102} - i^{104} + i^{106} - i^{108} + \ldots + i^{122}}_{11 \text{ pribrojnika}} = i^{4\cdot 25+2} - i^{4\cdot 26} + i^{4\cdot 26+2} - i^{4\cdot 27}$
$+ \ldots + i^{4\cdot 30+2} = -1 - 1 - 1 - 1 - \ldots - 1 = -11$;
4) $\underbrace{i^{101} - i^{103} + i^{105} - i^{107} + \ldots + i^{121}}_{11 \text{ pribrojnika}} = i^{4\cdot 25+1} - i^{4\cdot 25+3} + i^{4\cdot 26+1} - i^{4\cdot 26+3}$
$+ \ldots + i^{4\cdot 30+1} = i + i + i + i + \ldots + i = 11i$;
5) $1 + i^3 + i^6 + i^9 + i^{12} + i^{15} = 1 - i + (i^3)^2 + i^{4\cdot 2+1} + i^{4\cdot 3} + i^{4\cdot 3+3}$
$= 1 - i - 1 + i + 1 - i = 1 - i$;