

0.

Rješavanje rebusa

U svakom rebusu zadatak je isti: zamjeniti znak u tablici odgovarajućom znamenkom, tako da račun bude točan. Različitim znakovima uvijek odgovaraju različite znamenke. Znakovi predstavljaju znamenke broja, tako da prva znamenka broja nikad nije jednaka nuli.

Primjer 1.

$$\begin{array}{rcl}
 \equiv & \times & \blacksquare \times \blacksquare \\
 : & & + \\
 \times & + & \equiv \square \blacksquare = \equiv \square \blacksquare \\
 \hline
 \times & + & \blacksquare \square \blacksquare = \blacksquare \square \blacksquare
 \end{array}$$

Kad počnemo rješavati rebus, najprije treba dobro pogledati svaki njegov redak i stupac, ne bi li se odmah otkrio neki ključ. Na primjer: ako je zbroj dvaju jednoznamenkastih brojeva dvoznamenkast, onda je prva znamenka tog dvoznamenkastog broja jednaka 1. Ili: umnožak dvaju brojeva je jednoznamenkast. Broj takvih kombinacija je vrlo ograničen, pogotovo što ćemo vrlo lako uočiti ako je neki među njima jednak 1.

U ovom ćemo rebusu iskoristiti ove upute. Iz trećeg stupca odmah dobivamo prvu znamenku: $\blacklozenge = 1$. Iz prvog je stupca $\equiv = 4$ ili $\equiv = 9$. Ova druga mogućnost ne dolazi u obzir, jer bi umnožak broja 9 s brojem $\blacksquare \times$ bio veći od 199. Zato je $\equiv = 4$, pa onda i $\times = 2$.

Sad slijedi $\blacksquare = 3$ pa znamo prvi redak rebusa, a onda i cijeli rebus:

$$\begin{array}{r}
 4 \times 32 = 128 \\
 : + - \\
 2 + 46 = 48 \\
 \hline
 2 + 78 = 80
 \end{array}$$

Primjer 2.

$$\begin{array}{r}
 \equiv \times \blacksquare \times \equiv = \equiv \blacksquare \equiv \\
 : + - \\
 \blacksquare \diamond \blacksquare \times \blacksquare \blacksquare = \blacksquare \diamond \blacksquare \blacksquare \blacksquare \\
 \hline
 \blacksquare \times \blacksquare \equiv = \blacksquare \blacksquare \equiv
 \end{array}$$

Nakon pregleda cijele tablice zastajemo na posljednjem stupcu. $\equiv + \blacksquare = \equiv$ pa je $\blacksquare = 0$.

Sad gledamo prvi stupac. Broj \blacksquare pomnožen s brojem koji završava s \blacksquare ima završetak \blacksquare . Samo su tri broja sa tim svojstvom: 1, 5 i 6. 1 ne dolazi u obzir, a zbog prvog retka isključujemo i mogućnost $\blacksquare = 5$. Naime, 5 pomnožen s bilo kojim brojem daje broj koji završava s 5 ili s 0, a \equiv ne može biti niti 0 niti 5. To vidimo iz prvog retka tablice. Zato zaključujemo da mora biti $\blacksquare = 6$.

Umnožak brojeva u trećem retku je dvoznamenkast, zato mora biti $\blacksquare = 1$.

Nastavljamo s prvim retkom. Troznamenkasti broj $\equiv 16$ pomnožen s \equiv daje troznamenkasti broj. Zato je \equiv jednak 2 ili 3. Kako taj umnožak završava s \equiv , mora biti $\equiv = 2$. Rebus se sad raspliće do kraja:

$$\begin{array}{r}
 216 \times 2 = 432 \\
 : + - \\
 36 \times 10 = 360 \\
 \hline
 6 \times 12 = 72
 \end{array}$$

Primjer 3.

$$\begin{array}{rcl}
 \square\blacksquare\blacksquare & \times & \equiv = \square\blacksquare\blacklozenge\blacksquare\blacksquare \\
 + & + & : \\
 \blacksquare & \times & \blacksquare\blacksquare = \blacklozenge\blacksquare\equiv \\
 \hline
 \blacksquare\blacksquare - \blacklozenge\blacksquare\blacksquare & = & \blacklozenge\blacksquare\blacksquare
 \end{array}$$

Nakon pomnog pogleda u tablicu zaustaviti ćemo se na drugom stupcu. Zbroj jednoznamenkastih brojeva je dvoznamenkast, pa je zato $\blacksquare = 1$. Sad idemo na treći stupac. Umnožak dvaju dvoznamenkastih brojeva manjih od 20 je troznamenasti broj koji je manji od 400. Zato mora biti $\blacksquare = 2$ ili $\blacksquare = 3$. Promotrimo mogućnost $\blacksquare = 3$; troznamenasti broj u tablici tada je $31\square$. Trojka je onda prvi faktor u drugom retku, pa je posljednji broj u tom retku djeljiv s 3. Pomnožen s brojem $1\square$ daje $31\square$. Kako je $15 \times 19 < 300$, a $18 \times 17 = 306$ i $18 \times 18 = 324$, vidimo da ne možemo dobiti rješenje.

Zato mora biti $\blacksquare = 2$. Potražimo ostale brojeve.

Treći stupac sada mora glasiti $12 \times 18 = 216$, pa je $\equiv = 8$ i $\blacksquare = 6$. Rebus se sad raspliće sam od sebe:

$$\begin{array}{rcl}
 27 & \times & 8 = 216 \\
 + & + & : \\
 2 & \times & 9 = 18 \\
 \hline
 29 - 17 & = & 12
 \end{array}$$

Primjer 4.

$$\begin{array}{rcl}
 \blacksquare\blacksquare\equiv & \times & \blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare \\
 + & \times & : \\
 \equiv & - & \equiv\equiv\equiv\equiv\equiv\equiv \\
 \hline
 \equiv\equiv\equiv\equiv\equiv\equiv + \blacksquare\blacksquare\blacksquare\blacksquare & = & \blacklozenge\equiv\equiv\equiv\equiv\equiv\equiv
 \end{array}$$

Iz prvog retka čitamo da mora biti $\blacksquare \geq 3$, inače umnožak brojeva u prvom retku ne može premašiti 100.

Iz prvog stupca zaključujemo da je $\equiv = 1 + \blacksquare$, pa je $\equiv \geq 4$. Zato mora biti $\blacksquare \leq 5$, inače bi zbroj brojeva u trećem retku bio troznamenkast.

Ključ rješenja sada leži u drugom stupcu, u kojem se množe brojevi \blacksquare i $\blacksquare\blacksquare$. Što znamo o njima? To su dva uzastopna broja kojima umnožak ne završava znamenkom 0, a manji od njih iznosi barem 3. Tako u obzir dolazi samo par $(3, 4)$. Naime, par $(6, 7)$ već je prevelik, jer bi zbroj u trećem retku bio veći od 100.

Dakle, $\blacksquare = 3$, $\blacksquare\blacksquare = 4$. Ostatak se rebusa sada jednostavno dobiva. Najprije $\blacksquare\blacksquare = 1$, $\blacksquare = 2$ pa je onda i $\blacksquare\blacksquare\blacksquare = 6$, i račun je pri kraju. Evo rješenja:

$$\begin{array}{r} 36 \times 3 = 108 \\ + \quad \times \quad : \\ 6 - \quad 4 = \quad 2 \\ \hline 42 + 12 = \quad 54 \end{array}$$

Primjer 5.

$$\begin{array}{r} \blacksquare\blacksquare \times \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare \\ : \quad \quad \quad + \quad \quad \quad - \\ \blacksquare\blacksquare \times \blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare \\ \hline \blacksquare\blacksquare \times \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare \end{array}$$

Jednu je znamenku lako naći: $\blacksquare = 1$. Iz drugog retka $\blacksquare = 5$ ili $\blacksquare = 6$. Da ne može biti $\blacksquare = 5$ zaključujemo iz prvog stupca. Naime, onda bi moralo biti $\blacksquare = 0$, a to ne stoji zbog prvog retka. Zato je $\blacksquare = 6$, pa i $\blacksquare\blacksquare = 3$.

Sljedeći zaključak je $\blacksquare\blacksquare\blacksquare = 2$, jer zbrojen sa 6 daje jednoznamenkast broj (broj 3 je potrošen, a 1 ne dolazi u obzir).

Sad više ne preostaje mnogo mogućnosti za broj $\blacksquare\blacksquare$ u prvom retku. Lako je vidjeti da je $\blacksquare\blacksquare = 5$, $\blacksquare\blacksquare = 4$ i tako dobivamo rješenje:

$$\begin{array}{r} 54 \times 2 = 108 \\ : \quad \quad + \quad \quad - \\ 6 \times 6 = 36 \\ \hline 9 \times 8 = 72 \end{array}$$

* * *

Lako je uočiti da nema jednoznačnog puta u rješavanju ovih zadataka. U tome i leži njihova velika korist u razvijanju logičkog mišljenja. Također nikad nije moguće unaprijed kazati hoće li neki zadatak biti lagan ili težak.

Ako se rješenje rebusa ne nazire nakon temeljitog pregleda, korisno je napraviti zamjenu znakova.

Primjer 6.

$$\begin{array}{r}
 \begin{array}{c} \blacklozenge \blacksquare \\ \times \end{array} \quad \begin{array}{c} \square \blacksquare \\ + \end{array} \quad \begin{array}{c} \blacksquare \square \\ : \end{array} \quad \begin{array}{c} \blacksquare \blacksquare \blacksquare \\ - \end{array} \\
 \begin{array}{c} \blacksquare \blacksquare \blacksquare \\ \times \end{array} \quad \begin{array}{c} \square \square \\ = \end{array} \quad \begin{array}{c} \blacksquare \blacksquare \blacksquare \\ \hline \end{array} \\
 \hline
 \begin{array}{c} \blacksquare \blacksquare \blacksquare \\ + \end{array} \quad \begin{array}{c} \blacklozenge \\ = \end{array} \quad \begin{array}{c} \blacksquare \blacksquare \\ \hline \end{array}
 \end{array}$$

Napišimo je ovako:

$$\begin{array}{r}
 \begin{array}{c} CF \\ \times \\ G \\ + \\ DJ \\ \times \\ B \\ \hline HJ \\ + \\ C \end{array} = \begin{array}{c} AIF \\ : \\ - \\ JI \\ IB \end{array}
 \end{array}$$

Iz prvog stupca vidimo da je $F = 0$. Iz trećeg stupca slijedi $A = 1$.

Ključ rješenja je u drugom stupcu. Kako je $G = B \times C$, mora biti $G = 8$ ili 6. Pretpostavimo prvu mogućnost. Zbog $C0 \times 8 = 110$, mora biti $C = 2$, pa je $B = 4$. No to je kontradikcija s posljednjim retkom, jer iz $J + 2 = 4$ slijedi i $J = 2$. Zato mora biti $G = 6$.

Sada je $B = 2$, $C = 3$ ili $B = 3$, $C = 2$. Ova druga mogućnost otpada, jer bi iz trećeg retka bilo $J = 1$, a znamenka 1 već je potrošena. Zato mora biti $B = 2$, $C = 3$. Sada se lako razriješi cijeli rebus:

$$\begin{array}{r}
 \begin{array}{r}
 30 \times 6 = 180 \\
 + \quad : \quad - \\
 49 \times 2 = \quad 98 \\
 \hline
 79 + 3 = \quad 82
 \end{array}
 \end{array}$$

Primjer 7.

$$\begin{array}{r}
 \blacksquare \times \blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare \\
 + \quad + \quad : \\
 \blacksquare - \blacksquare = \blacksquare\blacksquare \\
 \hline
 \blacksquare + \blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare
 \end{array}$$

Prebacimo je u ružniji oblik:

$$\begin{array}{r}
 I \times HH = JHB \\
 + \quad + \quad : \\
 A - B = H \\
 \hline
 F + HA = II
 \end{array}$$

Iz prvog stupca postavljamo jednadžbu $A + E = F$. Iz posljednjeg retka: $F + E = A + 10$.

Eliminacijom nepoznanice A dobivamo

$$F + E = F - E + 10$$

pa je $E = 5$.

Sad je $B + D = 5$. Izbor $D = 1$ ili $D = 3$ nije moguć, jer je onda broj B paran, pa bi zbog trećeg stupca i D morao biti paran. Izbor $D = 4$ dao bi $B = 1$, što ne valja zbog trećeg stupca. Zato je $D = 2$, $B = 3$.

Sad je $I = 4$ i dobivamo rješenje:

$$\begin{array}{r}
 4 \times 33 = 132 \\
 + \quad + \quad : \\
 5 - 2 = 3 \\
 \hline
 9 + 35 = 44
 \end{array}$$

* * *

Izdvojimo još neke potpuno nasumice odabrane rebuse.

Primjer 8.

$$\begin{array}{rcl}
 \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ + \\ \blacksquare\blacksquare \end{array} & \times & \begin{array}{c} \blacksquare \\ \times \\ \blacksquare\blacksquare \end{array} = \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ \times \\ \blacksquare\blacksquare \end{array} : \\
 \hline
 \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ - \\ \blacksquare\blacksquare\blacksquare \end{array} & = & \begin{array}{c} \blacksquare \\ \blacksquare\blacksquare \\ \hline \blacksquare\blacksquare\blacksquare \end{array} = \begin{array}{c} \blacksquare \\ \blacksquare\blacksquare \\ \hline \blacksquare\blacksquare\blacksquare \end{array}
 \end{array}$$

Zbog množenja u prvom retku zaključujemo da je $\blacksquare = 1$. Također vrijedi $\blacksquare = 5$ ili $\blacksquare = 6$, jer broj pomnožen sam sa sobom ima istu posljednju znamenku.

Iz prvog stupca najprije slijedi $\blacksquare = 0$, a onda $\blacksquare = \blacksquare + 1$. Ta se dva broja množe u drugom stupcu. Mogućnost $\blacksquare = 5$, $\blacksquare = 6$ otpada, jer bi onda bilo $\blacksquare\blacksquare = 0$, što ne stoji. Dakle, vrijedi $\blacksquare = 6$, $\blacksquare = 7$ i rebus glasi:

$$\begin{array}{rcl}
 106 & \times & 6 = 636 \\
 & + & \times & : \\
 11 & - & 7 = & 4 \\
 \hline
 117 & + & 42 = 159
 \end{array}$$

Primjer 9.

$$\begin{array}{rcl}
 \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ \times \\ \blacksquare\blacksquare\blacksquare \end{array} & = & \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ \times \\ \blacksquare\blacksquare\blacksquare \end{array} : \\
 \hline
 \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ + \\ \blacksquare\blacksquare\blacksquare \end{array} & = & \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ \times \\ \blacksquare\blacksquare\blacksquare \end{array} \\
 \hline
 \begin{array}{c} \blacksquare\blacksquare\blacksquare \\ \times \\ \blacksquare\blacksquare\blacksquare \end{array} & - & \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare
 \end{array}$$

Pogledajmo drugi redak i drugi stupac. Broj $\blacksquare\blacksquare\blacksquare$ veći je ili jednak zbroju brojeva $\blacksquare\blacksquare\blacksquare$ i $\blacksquare\blacksquare\blacksquare$. Umnožak broja $\blacksquare\blacksquare\blacksquare$ s $\blacksquare\blacksquare\blacksquare$ veći je od 100. Zato je najmanja moguća vrijednost za $\blacksquare\blacksquare\blacksquare$ jednaka 4. Iz trećeg stupca zaključujemo da taj broj nije jednak 5. Dakle, on je ili jednak 4 ili veći od 5.

Pogledajmo sada treći stupac. Umnožak broja $\blacksquare\blacksquare\blacksquare$ s troznamenkastim brojem je troznamenkasti broj. Zato mora biti $\blacksquare = 1$, pa onda odmah $\blacksquare\blacksquare\blacksquare = 4$. Time saznajemo treći stupac: $576 : 144 = 4$. Prvi redak sad glasi $4\blacksquare\blacksquare \times 1\blacksquare = 576$ pa dobivamo $\blacksquare = 8$, $\blacksquare = 2$.

$$\begin{array}{r} 48 \times 12 = 576 \\ + \quad + \quad : \\ 4 \times 36 = 144 \\ \hline 52 - 48 = \quad 4 \end{array}$$

Primjer 10.

$$\begin{array}{rcl} \blacksquare\blacksquare\blacksquare \times \blacksquare\blacksquare\blacksquare\blacklozenge & = & \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare \\ + \quad \quad \quad : \quad \quad \quad - \\ \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare \times \blacksquare & = & \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare \\ \hline \blacksquare\blacksquare\blacksquare\blacklozenge\equiv & + & \equiv = \blacksquare\blacksquare\blacksquare\blacksquare\blacklozenge \end{array}$$

Zbog množenja u drugom retku odmah dobivamo $\blacksquare = 1$.

U zbrajanju u trećem retku imamo prijenos na mjestu desetica. Zato je $\equiv \geqslant 5$. Taj broj ne može biti 5, jer je $\blacklozenge \neq 0$, što se vidi iz trećeg stupca. Dakle, $\equiv \geqslant 6$.

Zbog zbrajanja u prvom stupcu, mora biti $\blacklozenge \geqslant 3$, zato ne dolazi u obzir niti $\equiv = 6$. (Pogledaj zbrajanje u trećem retku!). Zato mora biti $\equiv \geqslant 7$. Umnožak tog broja s \blacksquare daje broj manji od 20, pa je $\blacksquare = 2$.

Vratimo se na zbrajanje u prvom stupcu. Sigurno je $\blacklozenge \leqslant 4$, pa zbog drugog stupca slijedi da je $\blacklozenge = 4$, $\equiv = 7$.

$$\begin{array}{r} 28 \times 14 = 392 \\ + \quad \quad : \quad - \\ 119 \times 2 = 238 \\ \hline 147 + 7 = 154 \end{array}$$

Primjer 11.

$$\begin{array}{rcl}
 \blacksquare & \times & \blacksquare\blacksquare\blacksquare \\
 \times & & \times \\
 \blacksquare\blacksquare\blacksquare & + & \blacksquare\blacksquare\blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare\blacksquare\blacksquare & - & \blacksquare\blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare
 \end{array}$$

Jedino što se odmah može vidjeti jest $\blacksquare = 0$, zbog oduzimanja u trećem retku.

Iz prvog stupca vidimo da znamenka \blacksquare nije jednaka jedinici. Iz toga ćemo zaključiti da mora biti $\blacksquare = 1$. Naime, kad bi bilo $\blacksquare \geq 2$, onda bi, zbog množenja u drugom stupcu i zbrajanja u trećem, imali $\blacksquare \geq 6$ i $\blacksquare \geq 4$. To je nemoguće zbog trećeg retka; prvi bi broj morao biti veći od 1000. Dobili smo stoga $\blacksquare = 1$.

Sad je, zbog množenja u prvom stupcu, $\blacksquare \geq 7$. Kako iz trećeg stupca čitamo $\blacksquare + \blacksquare = 10$, posljednji broj u prvom retku je ili 128 ili 137. 137 je prost, pa prvi redak glasi: $8 \times 16 = 128$. Kad popunimo poznate brojeve u rebusu, dobit ćemo $\blacksquare = 9$, a onda i potpuno rješenje:

$$\begin{array}{rcl}
 8 \times 16 & = & 128 \\
 \times & \times & + \\
 113 + 39 & = & 152 \\
 \hline
 904 - 624 & = & 280
 \end{array}$$

Primjer 12.

$$\begin{array}{rcl}
 \blacksquare & \times & \blacksquare\blacksquare\blacksquare \\
 + & & + \\
 \blacksquare\blacksquare\blacksquare & - & \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare\blacksquare & + & \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare
 \end{array}$$

Iz trećeg retka slijedi $\blacksquare = 1$.

Razlika brojeva u drugom retku je jednoznamenkasta, zato je $\blacksquare = \blacksquare + 1$. Kako je ta razlika jednaka \blacksquare , mora biti $\blacksquare = 9$. Sada pogledajmo umnožak u trećem stupcu. Vidimo da mora biti $\blacksquare \geq 3$, pa onda i $\blacksquare \geq 4$. Inače, kad

bi bilo $\blacksquare = 2$, taj bi umnožak bio manji od 300, pa se znakovi \blacksquare i \blacksquare ne bi razlikovali.

Sada zaključujemo da vrijedi $\blacksquare \geq 6$, inače umnožak u prvom retku ne bi mogao premašiti 400. Zbog zbrajanja u drugom stupcu onda je $\blacksquare \leq 2$, pa je zato i $\blacksquare \leq 3$

Time dobivamo rješenje: $\blacksquare = 3$, $\blacksquare = 2$, $\blacksquare = 6$, a onda i ostatak:

$$\begin{array}{r} 6 \times 67 = 402 \\ + \quad + \quad : \\ 32 - 29 = \quad 3 \\ \hline 38 + 96 = 134 \end{array}$$

Primjer 13.

$$\begin{array}{rcl} \blacksquare \times \equiv\blacksquare + \equiv\blacklozenge & = & \blacksquare\blacksquare \\ + \quad - \quad \times & & + \\ \square\blacksquare\blacklozenge + \quad \equiv\equiv\equiv : \quad \blacksquare\blacksquare & = & \square\blacksquare\blacksquare \\ \hline \square\blacksquare\blacksquare \times \quad \blacksquare - \quad \blacksquare\blacksquare\blacksquare & = & \equiv\blacksquare\blacksquare\blacksquare \end{array}$$

Ovi su rebusi mnogo složeniji od prethodnih; teže je uočiti zakonitosti među brojevima.

Kao i prije, jedini brzi rezultat je $\equiv = 1$.

Zbog prijenosa u posljednjem stupcu zaključujemo da je $\blacksquare = 9$. Taj je podatak bitan za daljnju analizu.

Broj \equiv djeljiv je s \blacksquare . Zato mora biti $\blacksquare = 2$, $\blacksquare = 3$ ili $\blacksquare = 4$. Taj broj nije jednak jedinici, što vidimo iz trećeg stupca.

Ako je $\blacksquare = 2$, onda zbog prvog retka mora biti $\blacksquare = 7$, pa je iz drugog stupca $\equiv = 5$, što nije moguće, jer ovaj broj nije djeljiv s 2.

Ako je $\blacksquare = 3$, onda mora biti $\equiv = 6$, pa je iz drugog stupca $\equiv = 7$, što nije moguće, jer bi tada zbroj u prvom retku bio veći od 100.

Dakle, vrijedi $\blacksquare = 4$, pa je $\equiv = 8$, $\blacksquare = 6$. Jedina preostala mogućnost za \blacklozenge je 3, pa prvi redak glasi: $6 \times 14 + 13 = 97$. Treći stupac sad je $13 \times 4 = 52$, pa dobivamo rješenje cijelog rebusa. Drugi redak glasi $12 + 8 : 4 = 25$, a treći $29 \times 6 - 52 = 122$.

1.

Zagrijavanje

1.

$$\begin{array}{rcl}
 \blacksquare\equiv \times \equiv & = & \equiv\blacksquare = \equiv \times \blacksquare\equiv \\
 + \quad \times & : & + \quad + \\
 \equiv - \equiv & = & \equiv = \blacksquare - \blacksquare \\
 \hline
 \blacksquare\blacksquare + \blacksquare = \equiv\equiv & = & \blacklozenge + \blacksquare\blacksquare
 \end{array}$$

2.

$$\begin{array}{rcl}
 \blacksquare\equiv\equiv \times \blacksquare = \blacklozenge\blacksquare\blacksquare\blacksquare = \blacksquare\equiv\equiv \times \blacksquare \\
 + \quad \times & : & + \quad + \\
 \blacksquare - \equiv & = & \blacksquare = \blacksquare - \blacksquare \\
 \hline
 \blacksquare\blacksquare + \blacksquare = \blacksquare\equiv & = & \blacksquare\blacksquare + \blacksquare
 \end{array}$$

3.

$$\begin{array}{rcl}
 \equiv\blacksquare \times \equiv = \blacklozenge\blacksquare & = & \equiv \times \blacksquare\blacksquare \\
 + \quad \times & : & + \quad + \\
 \equiv - \blacksquare & = & \blacksquare = \blacksquare - \blacksquare \\
 \hline
 \blacksquare\equiv + \blacksquare\blacksquare = \blacksquare\blacksquare & = & \equiv\blacksquare + \blacksquare\blacksquare
 \end{array}$$

4.

$$\begin{array}{rcl}
 \blacklozenge \times \blacksquare\blacksquare = \blacksquare\blacksquare & = & \equiv \times \equiv\blacksquare \\
 + \quad \times & : & + \quad + \\
 \blacksquare - \blacksquare & = & \blacksquare = \equiv - \blacklozenge \\
 \hline
 \blacksquare + \equiv\blacksquare = \blacksquare\blacklozenge & = & \blacksquare\blacksquare + \equiv\blacklozenge
 \end{array}$$

5.

$$\begin{array}{rcl}
 \blacklozenge\blacksquare & \times & \equiv = \blacksquare\blacksquare\blacksquare = \blacklozenge & \times & \equiv\equiv \\
 + & & \times & : & + \\
 \blacksquare - & \blacksquare\blacksquare = & \blacksquare\blacksquare = & \blacksquare - & \blacksquare\blacksquare \\
 \hline
 \blacklozenge\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare = & \blacksquare\blacklozenge\blacksquare = & \blacksquare\blacksquare\blacksquare\blacksquare + \equiv\blacksquare\blacksquare
 \end{array}$$

6.

$$\begin{array}{rcl}
 \blacksquare\blacklozenge & \times & \blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare = \blacksquare & \times & \blacksquare\blacklozenge \\
 + & & \times & : & + \\
 \blacksquare - & \blacksquare\blacksquare\blacksquare = & \equiv = & \blacklozenge - & \blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare = & \equiv\blacksquare\blacksquare = & \blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare\blacksquare
 \end{array}$$

7.

$$\begin{array}{rcl}
 \blacksquare\equiv & \times & \blacksquare = \blacksquare\blacksquare\blacksquare = \blacksquare\equiv & \times & \blacksquare \\
 + & & \times & : & + \\
 \blacksquare\blacksquare - & \blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare = & \equiv - & \blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare
 \end{array}$$

8.

$$\begin{array}{rcl}
 \equiv\blacksquare & \times & \blacksquare\blacksquare = \blacksquare\blacksquare = \blacksquare\blacksquare & \times & \blacksquare\blacksquare \\
 + & & \times & : & + \\
 \blacksquare\blacksquare - & \blacksquare\blacksquare = & \blacksquare\blacksquare = & \blacksquare\blacksquare - & \blacksquare\blacksquare \\
 \hline
 \blacksquare\equiv + \blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare + \blacklozenge
 \end{array}$$

9.

$$\begin{array}{rcl}
 \blacksquare & \times & \blacklozenge = \blacksquare\blacksquare\blacksquare = \blacksquare & \times & \blacksquare\blacksquare \\
 + & & \times & : & + \\
 \blacksquare\blacksquare - & \blacksquare\blacksquare = & \blacksquare\blacksquare = & \blacksquare\blacksquare - & \blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare
 \end{array}$$

10.

$$\begin{array}{rcl}
 \blacksquare\blacksquare\blacksquare & \times & \blacksquare\blacklozenge = \blacksquare\blacksquare\blacksquare\blacksquare = \blacklozenge\blacksquare & \times & \blacksquare\blacksquare \\
 + & & \times & : & + \\
 \blacksquare\blacksquare - & \blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare - & \blacksquare\blacksquare \\
 \hline
 \blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare = & \blacksquare\blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare
 \end{array}$$

11.

$$\begin{array}{rcl}
 \times \square \times \times \square \square = \square \square \diamond \times \square & = & \square \square \times \times \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \diamond - \square \square = \square \square = \square \square \square \square - \square \square \\
 \hline
 \square \square + \square \square \square = \square \square \square = \square \square \square + \square \square \square
 \end{array}$$

12.

$$\begin{array}{rcl}
 \square \square \square \times \square \square \square = \square \square \square \square \square = \square \square \square \times \square \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \square \square \square - \square \square = \diamond \square = \square \square \square \square - \square \square \\
 \hline
 \square \square \square + \square \square \square = \diamond \square = \square \square \square + \square \square \square
 \end{array}$$

13.

$$\begin{array}{rcl}
 \square \square \square \times \square \square \times = \square \square \square \times = \square \square \square \times \square \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \square \square - \square \square = \square \square = \square \square \diamond \square - \square \square \\
 \hline
 \diamond \square + \square \square \times = \square \square \square = \diamond \square \square + \square \square \square
 \end{array}$$

14.

$$\begin{array}{rcl}
 \square \square \square \times \square \square \times = \square \square \square \square = \square \square \times \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \square \square - \square \square = \square \square = \square \square \square \square - \square \square \square \\
 \hline
 \square \square + \square \square \square = \square \square = \square \square \square + \square \square \square
 \end{array}$$

15.

$$\begin{array}{rcl}
 \square \square \times \square \square = \square \square \square = \square \square \times \square \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \square \square - \square \square = \square \square = \square \square \square - \square \square \\
 \hline
 \square \square \square + \square \square = \square \square \square = \square \square \square + \square \square \square
 \end{array}$$

16.

$$\begin{array}{rcl}
 \square \square \times \square \square = \square \square \square \square = \square \square \times \square \square \square \\
 + \quad \quad \quad \times \quad \quad \quad : \quad \quad \quad + \quad \quad \quad + \\
 \square \square \square - \square \square = \square \square = \square \square \square \diamond - \square \square \\
 \hline
 \square \square + \square \square \square = \square \square \square \square = \square \square \square + \square \square \square
 \end{array}$$

17.

$$\begin{array}{rcl}
 \text{■■■} \times \text{■■} & = & \text{■■■■■} \\
 + & \times & : \\
 \text{■■■} - \text{■■} & = & \text{■■■} = \text{■■■} - \text{■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■■} \times \text{■■■} & = & \text{■■} \\
 + & + & \\
 \hline
 \text{■■■} + \text{■■■} & = & \text{■■■■■} = \text{■■■} + \text{■■■}
 \end{array}$$

18.

$$\begin{array}{rcl}
 \text{■◆} \times \text{■} & = & \text{■■■■■} \\
 + & \times & : \\
 \text{■■■} - \text{■} & = & \text{■} = \text{■■■} - \text{■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■◆} \times \text{■■■■■} & = & \text{■} \\
 + & + & \\
 \hline
 \text{■◆} + \text{■■■■■} & = & \text{■■■■■} = \text{■■■■■} + \text{■■■■■}
 \end{array}$$

19.

$$\begin{array}{rcl}
 \text{■■■} \times \text{■■■} & = & \text{◆◆■■} \\
 + & \times & : \\
 \text{■■} - \text{■} & = & \text{■} = \text{■■■} - \text{■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■■■} + \text{■■■} & = & \text{■■■} = \text{■■■} + \text{■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■} \times \text{■■■} & = & \text{■■■} \\
 + & + & \\
 \hline
 \text{■■■} + \text{■■■} & = & \text{■■■} = \text{■■■} + \text{■■■}
 \end{array}$$

20.

$$\begin{array}{rcl}
 \text{■■■} \times \text{■■■■■} & = & \text{■◆■■■} \\
 + & \times & : \\
 \text{■■■■■} - \text{■■■} & = & \text{■} = \text{■■■■■} - \text{■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■■■} + \text{■■■} & = & \text{■■■} = \text{■■■} + \text{■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■■■■■} \times \text{■■■} & = & \text{■■■} \\
 + & + & \\
 \hline
 \text{■■■■■} + \text{■■■} & = & \text{■■■■■} = \text{■■■■■} + \text{■■■}
 \end{array}$$

21.

$$\begin{array}{rcl}
 \text{■■} \times \text{■◆} & = & \text{■■■■} \\
 + & \times & : \\
 \text{■■} - \text{■} & = & \text{■} = \text{■■■■} - \text{■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■■■■} + \text{■} & = & \text{■■■■} = \text{■■■■} + \text{■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■◆} \times \text{■■■■} & = & \text{■■■■} \\
 + & + & \\
 \hline
 \text{■■■■} + \text{■■■■} & = & \text{■■■■} = \text{■■■■} + \text{■■■■}
 \end{array}$$

22.

$$\begin{array}{rcl}
 \text{■} \times \text{■■■■■} & = & \text{■◆■■■■} \\
 + & \times & : \\
 \text{■■■■■} - \text{■} & = & \text{■■■■■} = \text{■■■■■} - \text{■■■■■}
 \end{array}
 \quad
 \begin{array}{rcl}
 \text{■} \times \text{■■■■■} & = & \text{■■■■■} \\
 + & + & \\
 \hline
 \text{■■■■■} + \text{■■■■■} & = & \text{■■■■■} = \text{■■■■■} + \text{■■■■■}
 \end{array}$$

23.

$$\begin{array}{rcl}
 \blacklozenge \times \boxtimes\blacksquare & = & \boxtimes\blacksquare\blacksquare \\
 + \times & : & + + \\
 \blacksquare - \blacksquare & = & \blacksquare = \blacklozenge - \equiv \\
 \hline
 \boxtimes\blacksquare + \blacksquare\blacksquare & = & \boxtimes\blacksquare = \boxtimes\blacksquare + \blacksquare\blacksquare
 \end{array}$$

24.

$$\begin{array}{rcl}
 \blacksquare \times \blacksquare & = & \equiv\blacksquare \\
 + \times & : & + + \\
 \equiv - \blacksquare & = & \blacksquare = \blacksquare - \equiv \\
 \hline
 \boxtimes + \boxtimes & = & \boxtimes = \boxtimes + \boxtimes
 \end{array}$$

25.

$$\begin{array}{rcl}
 \blacksquare\blacksquare\blacksquare \times \boxtimes\blacksquare & = & \boxtimes\blacksquare\blacklozenge \\
 + \times & : & + + \\
 \blacksquare - \blacksquare & = & \blacksquare = \boxtimes\blacksquare - \blacksquare \\
 \hline
 \boxtimes\blacksquare + \boxtimes\blacksquare & = & \blacksquare\blacksquare = \boxtimes\blacksquare + \boxtimes\blacksquare
 \end{array}$$

26.

$$\begin{array}{rcl}
 \blacksquare\blacksquare \times \boxtimes\blacksquare & = & \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare \times \blacksquare\blacksquare \\
 + \times & : & + + \\
 \blacksquare - \blacksquare & = & \blacklozenge = \boxtimes\blacksquare - \blacklozenge \\
 \hline
 \blacksquare\blacksquare + \blacksquare\blacksquare & = & \blacksquare\blacksquare = \boxtimes\blacksquare + \boxtimes\blacklozenge
 \end{array}$$

27.

$$\begin{array}{rcl}
 \equiv\blacksquare \times \boxtimes\blacklozenge & = & \equiv\blacksquare\blacklozenge \\
 + \times & : & + + \\
 \boxtimes\blacksquare - \blacksquare & = & \blacksquare = \boxtimes\blacksquare - \equiv \\
 \hline
 \equiv\blacksquare + \boxtimes\blacklozenge & = & \boxtimes\blacksquare\equiv = \boxtimes\blacksquare\blacksquare + \boxtimes\blacksquare
 \end{array}$$

28.

$$\begin{array}{rcl}
 \boxtimes\blacksquare \times \blacksquare\blacksquare & = & \blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare \times \boxtimes\blacksquare \\
 + \times & : & + + \\
 \blacksquare\blacksquare - \boxtimes & = & \blacksquare = \blacksquare\blacksquare - \blacklozenge \\
 \hline
 \equiv\blacksquare + \blacksquare\blacksquare & = & \boxtimes\blacksquare = \blacksquare\blacksquare + \blacksquare\blacksquare
 \end{array}$$

29.

$$\begin{array}{rcl}
 \times \square \square & \times & \square = \square \square \square = \times \square \square & \times & \square \\
 + & \times & : & + & + \\
 \square - \diamond = & \square = & \times \square - \times \square \\
 \hline
 \times \square \diamond + \square \diamond = & \times \square \square \square = & \times \square \square \square + \times \square \square
 \end{array}$$

30.

$$\begin{array}{rcl}
 \times \square \diamond & \times \square \square \square = & \square \square \square = \square \square \square \times \square \square \\
 + & \times & : & + & + \\
 \square - \times = & \diamond = & \square \square - \square \square \\
 \hline
 \square \square + \square \square = & \square \square = & \square \square + \times \square \square
 \end{array}$$

31.

$$\begin{array}{rcl}
 \square \square \diamond & \times \square \square = & \square \square \square \times = \square \times \square \square \square \\
 + & \times & : & + & + \\
 \square - \square = & \square \square = & \square - \square \square \\
 \hline
 \square \square + \square \square = & \square \square = & \square \square + \square \square \square
 \end{array}$$

32.

$$\begin{array}{rcl}
 \square \square \square & \times \diamond \square = & \times \square \square \square = \square \square \times \diamond \square \\
 + & \times & : & + & + \\
 \square - \square = & \square = & \times \square \square - \square \square \\
 \hline
 \square \square \square + \square \square \square = & \square \square \square = & \square \square \times + \square \square \square
 \end{array}$$

33.

$$\begin{array}{rcl}
 \square \square \square & \times \square \square = & \square \square \square \times = \square \square \times \square \square \\
 + & \times & : & + & + \\
 \square \square \square - \square \square = & \square \square = & \square \square \square - \square \square \\
 \hline
 \square \square + \square \square \square = & \square \square \square = & \diamond \square \square + \times \square \square
 \end{array}$$

34.

$$\begin{array}{rcl}
 \square \square \diamond & \times \square \diamond = & \square \square \square \times = \times \square \square \times \square \square \\
 + & \times & : & + & + \\
 \diamond - \times = & \times = & \times \square \square - \times \square \square \\
 \hline
 \square \square \diamond + \square \square \square = & \square \square \times = & \diamond \square \square + \square \square \square
 \end{array}$$

35.

$$\begin{array}{ccccccccc}
 \begin{array}{c} \square \blacksquare \\ \times \\ + \\ \blacksquare \end{array} & \begin{array}{c} \blacksquare \\ = \\ \times \\ - \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \blacksquare \\ = \\ : \\ \blacksquare \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \\ = \\ + \\ \blacksquare \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \\ \times \\ + \\ - \end{array} \\
 \end{array}$$

$$\begin{array}{ccccccccc}
 \begin{array}{c} \blacksquare \blacksquare \square \\ + \\ \blacksquare \blacksquare \square \end{array} & \begin{array}{c} = \\ = \\ = \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \blacksquare \\ = \\ \blacksquare \blacksquare \square \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \\ + \\ \blacksquare \blacksquare \square \end{array} & \begin{array}{c} \blacksquare \blacksquare \square \\ + \\ \blacksquare \blacksquare \square \end{array} \\
 \end{array}$$

36.

$$\begin{array}{ccccccccc}
 \square\blacksquare & \times & \blacksquare & = & \blacksquare\blacklozenge\blacksquare & = & \blacksquare & \times & \square\blacksquare \\
 + & & \times & & : & & + & + & + \\
 \blacksquare\blacksquare & - & \blacksquare & = & \blacksquare & = & \blacksquare\blacksquare & - & \blacksquare\blacksquare\blacksquare
 \end{array}$$

$$\blacksquare\blacklozenge + \blacklozenge\blacksquare = \blacksquare\blacksquare\blacksquare\blacksquare\blacksquare = \blacksquare\blacksquare\blacksquare + \blacksquare\blacksquare\blacksquare$$

37.

$$\begin{array}{ccccccc}
 \boxed{\text{---}} & \times & \boxed{\text{---}} & = & \boxed{\text{---}} & \times & \boxed{\text{---}} \\
 + & & \times & & : & + & + \\
 \boxed{\text{---}} & - & \boxed{\text{---}} & = & \boxed{\text{---}} & = & \boxed{\text{---}} \\
 \hline
 \boxed{\text{---} \blacklozenge} & + & \boxed{\text{---}} & = & \boxed{\text{---} \blacklozenge} & = & \boxed{\text{---}} & + & \boxed{\text{---}}
 \end{array}$$

38.

$$\begin{array}{ccccccccc}
 \begin{array}{c} \times \\ + \\ - \end{array} & \times & \begin{array}{c} \square \blacksquare \\ \times \\ \diamondsuit \end{array} & = & \begin{array}{c} \times \\ : \\ \square \blacksquare \end{array} & = & \begin{array}{c} \square \blacksquare \\ + \\ - \end{array} & \times & \begin{array}{c} \square \blacksquare \\ + \\ \blacksquare \end{array} \\
 \hline
 \begin{array}{c} \blacksquare \blacksquare \blacksquare \blacksquare \\ + \\ - \end{array} & + & \begin{array}{c} \diamondsuit \blacksquare \\ \times \\ \blacksquare \end{array} & = & \begin{array}{c} \square \blacksquare \\ : \\ \blacksquare \end{array} & = & \begin{array}{c} \square \blacksquare \blacksquare \blacksquare \blacksquare \\ + \\ - \end{array} & - & \begin{array}{c} \blacksquare \blacksquare \blacksquare \blacksquare \\ + \\ \blacksquare \end{array}
 \end{array}$$

39.

$$\begin{array}{ccccccccc}
 \text{[1]} & \times & \text{[2]} & = & \text{[3]} & \times & \text{[4]} & \times & \text{[5]} \\
 + & & \times & & : & & + & & + \\
 \text{[1]} & - & \text{[2]} & = & \text{[3]} & = & \text{[4]} & - & \text{[5]}
 \end{array}$$

$$\text{[1]} \text{[2]} + \text{[3]} \text{[4]} = \text{[5]} \text{[6]} = \text{[7]} \text{[8]} + \text{[9]} \text{[10]}$$

40.

$$\begin{array}{ccccccccc}
 \boxtimes & \times & \blacksquare \blacksquare & = & \blacklozenge \blacksquare \blacksquare \blacksquare & = & \blacksquare \blacksquare & \times & \boxtimes \blacksquare \blacksquare \blacksquare \\
 + & & \times & & : & & + & + & + \\
 \blacksquare & - & \blacksquare \blacksquare & = & \blacksquare \blacksquare & = & \blacksquare \blacksquare \blacksquare & - & \blacksquare \blacksquare \blacksquare \\
 \hline
 \blacksquare \blacksquare \blacksquare & + & \blacksquare \blacksquare \blacksquare & = & \blacksquare \blacksquare \blacksquare & = & \blacksquare \blacksquare \blacksquare & + & \boxtimes \blacklozenge \blacksquare \blacksquare
 \end{array}$$