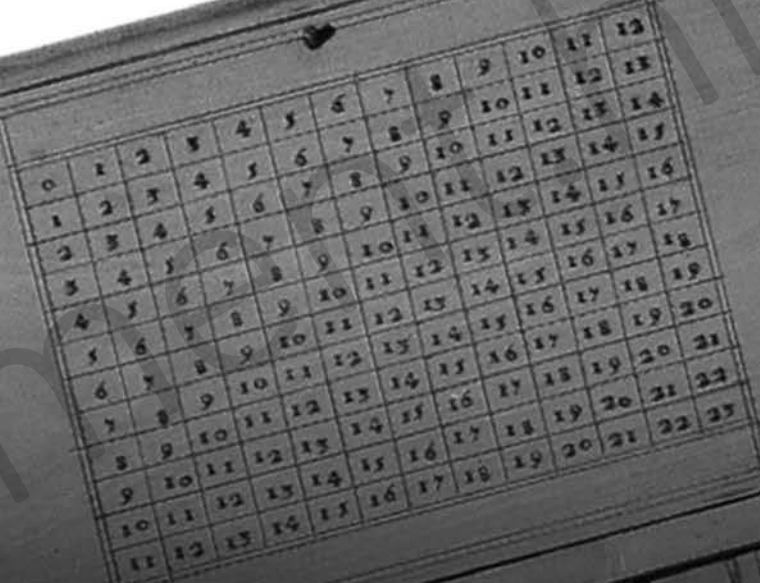


1

Korijeni i potencije



0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	3	4	5	6	7	8	9	10	11	12	13	14	15
3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	5	6	7	8	9	10	11	12	13	14	15	16	17
5	6	7	8	9	10	11	12	13	14	15	16	17	18
6	7	8	9	10	11	12	13	14	15	16	17	18	19
7	8	9	10	11	12	13	14	15	16	17	18	19	20
8	9	10	11	12	13	14	15	16	17	18	19	20	21
9	10	11	12	13	14	15	16	17	18	19	20	21	22
10	11	12	13	14	15	16	17	18	19	20	21	22	23
11	12	13	14	15	16	17	18	19	20	21	22	23	

1.1. Korijeni

Zadatak 1. Koliko je:

$$1) 49^{\frac{1}{2}}; \quad 2) -0.09^{\frac{1}{2}}; \quad 3) \left(\frac{16}{25}\right)^{\frac{1}{2}}; \quad 4) (-144)^{\frac{1}{2}}?$$

Rješenje. 1) $49^{\frac{1}{2}} = \sqrt{49} = 7$; 2) $-0.09^{\frac{1}{2}} = -\sqrt{0.09} = -0.3$;

3) $\left(\frac{16}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$; 4) $(-144)^{\frac{1}{2}}$ nema realnih rješenja.

Zadatak 2. Izračunaj:

$$1) -27^{\frac{1}{3}}; \quad 2) \left(\frac{1}{8}\right)^{\frac{1}{3}}; \quad 3) \left(-\frac{1}{8}\right)^{\frac{1}{3}}; \quad 4) \left(-\frac{64}{125}\right)^{\frac{1}{3}};$$

$$5) 16^{\frac{1}{4}}; \quad 6) -16^{\frac{1}{4}}; \quad 7) \left(\frac{81}{625}\right)^{\frac{1}{4}}; \quad 8) (-256)^{\frac{1}{4}}.$$

Rješenje. 1) $-27^{\frac{1}{3}} = -\sqrt[3]{27} = -3$; 2) $\left(\frac{1}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$;

3) $\left(-\frac{1}{8}\right)^{\frac{1}{3}} = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$; 4) $\left(-\frac{64}{125}\right)^{\frac{1}{3}} = \sqrt[3]{-\frac{64}{125}} = -\frac{4}{5}$;

5) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$; 6) $-16^{\frac{1}{4}} = -\sqrt[4]{16} = -2$;

7) $\left(\frac{81}{625}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{81}{625}} = \frac{3}{5}$; 8) $(-256)^{\frac{1}{4}}$ nema realnih rješenja.

Zadatak 3. Izračunaj:

$$1) -32^{\frac{1}{5}}; \quad 2) (-32)^{\frac{1}{5}}; \quad 3) 243^{\frac{1}{5}}; \quad 4) \left(\frac{1}{64}\right)^{\frac{1}{6}};$$

$$5) 2^{\frac{1}{4}}; \quad 6) 3^{\frac{1}{3}}; \quad 7) (-10)^{\frac{1}{3}}; \quad 8) 16^{\frac{1}{5}}.$$

Rješenje. 1) $-32^{\frac{1}{5}} = -\sqrt[5]{32} = -2$; 2) $(-32)^{\frac{1}{5}} = \sqrt[5]{-32} = -2$;

3) $243^{\frac{1}{5}} = \sqrt[5]{243} = 3$; 4) $\left(\frac{1}{64}\right)^{\frac{1}{6}} = \sqrt[6]{64} = 2$;

5) $2^{\frac{1}{4}} = \sqrt[4]{2} = 1.189207$; 6) $3^{\frac{1}{3}} = \sqrt[3]{3} = 1.44225$;

7) $(-10)^{\frac{1}{3}} = \sqrt[3]{-10} = -\sqrt[3]{10} = 2.154435$; 8) $16^{\frac{1}{5}} = \sqrt[5]{16} = 1.741101$.

Zadatak 4. Izračunaj:

$$1) \sqrt[4]{16}; \quad 2) \sqrt[4]{-16}; \quad 3) \sqrt[3]{-\frac{27}{125}}; \quad 4) \sqrt[5]{32};$$

$$5) \sqrt[5]{-32}; \quad 6) \sqrt[4]{\frac{16}{81}}; \quad 7) \sqrt[6]{64}; \quad 8) \sqrt[3]{-343}.$$

- Rješenje.** 1) $\sqrt[4]{16} = 2$; 2) $\sqrt[4]{-16}$ nema realnih rješenja; 3) $\sqrt[3]{-\frac{27}{125}} = -\frac{3}{5}$;
 4) $\sqrt[5]{32} = 2$; 5) $\sqrt[5]{-32} = -2$; 6) $\sqrt[4]{\frac{16}{81}} = \frac{2}{3}$;
 7) $\sqrt[6]{64} = 2$; 8) $\sqrt[3]{-343} = -7$.

Zadatak 5. Riješi u skupu realnih brojeva jednažbe:

- 1) $x^3 = 125$; 2) $8x^3 = 27$; 3) $16x^4 - 1 = 0$;
 4) $81x^4 - 256 = 0$; 5) $x^5 = 16$; 6) $3x^4 = 25$.

- Rješenje.** 1) $x^3 = 125$, $x = \sqrt[3]{125}$, $x = 5$; 2) $8x^3 = 27$, $x = \sqrt[3]{\frac{27}{8}}$, $x = \frac{3}{2}$;
 3) $16x^4 - 1 = 0$, $16x^4 = 1$, $x^4 = \frac{1}{16}$, $x = \sqrt[4]{\frac{1}{16}}$, $x_1 = \frac{1}{2}$, $x_2 = -\frac{1}{2}$;
 4) $81x^4 - 256 = 0$, $81x^4 = 256$, $x^4 = \frac{256}{81}$, $x = \sqrt[4]{\frac{256}{81}}$, $x_1 = \frac{4}{3}$, $x_2 = -\frac{4}{3}$;
 5) $x^5 = 16$, $x = \sqrt[5]{16}$, $x = 1.741101$;
 6) $3x^4 = 25$, $x^4 = \frac{25}{3}$, $x = \sqrt[4]{\frac{25}{3}}$, $x_1 = 1.699044$, $x_2 = -1.699044$.

Zadatak 6. Područje koje promatramo mikroskopom ima površinu od $3 \cdot 10^{-5} \text{ m}^2$. Na stakalcu se nalazi okrugao predmet promatranja koji pokriva otprilike četvrtinu površine. Koliki je njegov promjer?

- Rješenje.** Izrazimo površinu u centimetrima kvadratnim, $3 \cdot 10^{-5} \text{ m}^2 = 3 \cdot 10^{-1} \text{ cm}^2 = 0.3 \text{ cm}^2$. Ako predmet promatranja prekriva $\frac{1}{4}$ površine, tad je površina tog predmeta $\frac{1}{4} \cdot 0.3 = \frac{3}{40} \text{ cm}^2$. Predmet je okrugao pa se njegova površina računa formulom $P = r^2\pi$ pri čemu je r polumjer predmeta. Dakle, $\frac{3}{40} = r^2\pi$, odnosno $r^2 = \frac{3}{40\pi}$. Dobivamo $r = \sqrt{\frac{3}{40\pi}} \approx 0.1545$ pa je promjer promatranog tijela $2 \cdot 0.1545 = 0.309 \text{ cm}$.

Zadatak 7. Geometrijska sredina n pozitivnih brojeva a_1, \dots, a_n definira se formulom

$$G = \sqrt[n]{a_1 \cdot a_2 \cdot \dots \cdot a_n}$$

Izračunaj sredinu sljedećih brojeva i usporedi je s aritmetičkom sredinom istih brojeva. Što uočavaš?

- 1) 8, 9;
 2) 8, 8, 9, 9;
 3) 1, 2, 3, 4, 5, 100;
 4) 13.1, 13.4, 13.2, 13.8, 13.3, 12.9, 13.6.

- Rješenje.** Aritmetička sredina n pozitivnih brojeva a_1, \dots, a_n definira se formulom $A = \frac{a_1 + a_2 + \dots + a_n}{n}$. Uvrštavanjem zadanih brojeva dobivamo:

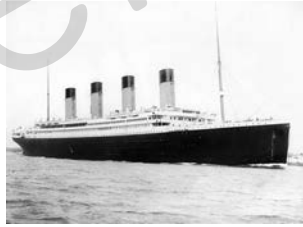
$$1) G = \sqrt{72} \approx 8.485, A = \frac{17}{2} = 8.5; \quad 2) G = \sqrt[4]{5184} \approx 8.485, \\ A = \frac{34}{4} = 8.5; \quad 3) G = \sqrt[6]{12\,000} \approx 4.785, A = \frac{115}{6} = 19.167; \\ 4) G \approx 13.326, A = 13.329.$$

Usvim primjerima aritmetička sredina je veća od geometrijske.

Zadatak 8.

Formulom $v = 6.5p^{\frac{1}{7}}$ izražava se ovisnost brzine broda u čvorovima o snazi p brodskog motora u konjskim snagama (1 čvor = 1.15 mi/h = 1.85 km/h).

- 1) Kolikom se brzinom kreće brod čiji motor ima snagu od 600 KS?
- 2) Ako se snaga motora udvostruči, kojom će se brzinom kretati brod?



- 3) Brzina Titanika pri udaru o santu bila je 18.5 čv. Kolikom su snagom prije isključenja radili motori?

Rješenje.

$$1) v = 6.5 \cdot 600^{\frac{1}{7}} = 6.5 \cdot 2.4939 = 16.21 \text{ čv}; \\ 2) v = 6.5 \cdot (2 \cdot 600)^{\frac{1}{7}} = 6.5 \cdot 1200^{\frac{1}{7}} = 6.5 \cdot 2.75 = 17.9 \text{ čv}; \\ 3) p^{\frac{1}{7}} = \frac{18.5}{6.5} = 2.846, p = 1513 \text{ KS}.$$

Zadatak 9.

Flora Malezije jedna je od najraznovrsnijih na Zemlji. Dvije trećine kopnene površine pokriveno je šumama s oko 2000 različitih vrsta stabala. Na jednom hektaru može se pronaći i do 240 različitih vrsta. Vidi na poveznici https://en.wikipedia.org/wiki/List_of_plants_of_Malaysia. Prosječan broj vrsta drveća u području površine P (u kvadratnim miljama) može se prikazati formulom

$$N = 435 P^{\frac{1}{4}}$$

koja je dobra aproksimacija za područja umjerene veličine (par desetaka kvadratnih milja).

Odredi očekivani broj vrsta drveća u području površine 50 km^2 .

Rješenje.

Kako bismo mogli iskoristiti formulu, potrebno je kvadratne kilometre pretvoriti u kvadratne milje. Vrijedi: 1 kvadratna milja = 2.59 km^2 pa je

$$50 \text{ km}^2 = \frac{50}{2.59} = 19.31 \text{ kvadratnih milja. Sada računamo}$$

$$N = 435 \cdot 19.31^{\frac{1}{4}} \approx 435 \cdot 2.096 = 911.82.$$

Očekivan broj vrsta drveća je 912.

1.2. Potencije s racionalnim eksponentom

Zadatak 1. Zapiši u obliku potencije:

- 1) $\sqrt[3]{4}$; 2) $\sqrt[4]{27}$; 3) $\frac{1}{\sqrt{10}}$;
 4) $\sqrt[5]{16}$; 5) $\frac{1}{\sqrt[3]{12}}$; 6) $\sqrt[4]{0.01^3}$;
 7) $\sqrt[4]{a^2 - b^2}$; 8) $\frac{1}{\sqrt{a+b}}$.

Rješenje. 1) $\sqrt[3]{4} = \sqrt[3]{2^2} = 2^{\frac{2}{3}}$; 2) $\sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}$;
 3) $\frac{1}{\sqrt{10}} = (\sqrt{10})^{-1} = 10^{-\frac{1}{2}}$; 4) $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$.

Zadatak 2. Zapiši u obliku korijena sljedeće potencije:

- 1) $3^{\frac{2}{3}}$; 2) $8^{\frac{3}{4}}$; 3) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$;
 4) $25^{-0.25}$; 5) $0.1^{-\frac{1}{2}}$; 6) $\frac{1}{(a-b)^{-\frac{1}{3}}}$;
 7) $(a^2 + b^2)^{\frac{3}{2}}$; 8) $(a^4 + b^4)^{\frac{1}{4}}$.

Rješenje. 1) $\sqrt[3]{3^2} = \sqrt[3]{9}$; 2) $\sqrt[4]{8^3} = \sqrt[4]{512}$; 3) $\sqrt{\frac{1}{4}}$; 4) $\left(\frac{1}{25}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{1}{25}}$;
 5) $10^{\frac{1}{2}} = \sqrt{10}$; 6) $(a-b)^{\frac{1}{3}} = \sqrt[3]{a-b}$; 7) $\sqrt{(a^2 + b^2)^3}$; 8) $\sqrt[4]{a^4 + b^4}$.

Zadatak 3. Između koja se dva uzastopna cijela broja nalaze sljedeći brojevi:

- 1) $100^{\frac{1}{3}}$; 2) $8^{\frac{3}{4}}$; 3) $\left(\frac{1}{2}\right)^{-\frac{5}{2}}$; 4) $-0.1^{-1.5}$?

Rješenje. 1) 4 i 5. 2) 4 i 5; 3) 2 i 3; 4) -32 i -31.

Zadatak 4. Izračunaj:

- 1) $16^{-\frac{1}{2}} - 0.25^{1.5}$; 2) $0.008^{-\frac{2}{3}} \cdot \left(\frac{1}{25}\right)^{-0.5}$;
 3) $0.04^{-1.5} \cdot \left(\frac{1}{125}\right)^{\frac{2}{3}}$; 4) $\left(\frac{1}{9}\right)^{0.5} + 0.027^{-\frac{2}{3}}$.

Rješenje. 1) $16^{-\frac{1}{2}} - 0.25^{1.5} = \sqrt{\frac{1}{16}} - \left(\frac{1}{4}\right)^{1.5} = \frac{1}{4} - \left(\frac{1}{2}\right)^3 = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$;
 2) $0.008^{-\frac{2}{3}} \cdot \left(\frac{1}{25}\right)^{-0.5} = (0.2)^{-3 \cdot \frac{2}{3}} \cdot \left(\frac{1}{25}\right)^{-\frac{1}{2}} = \left(\frac{1}{5}\right)^{-2} \cdot \sqrt{25} = 25 \cdot 5 = 125$;

$$3) 0.04^{-1.5} \cdot \left(\frac{1}{125}\right)^{\frac{2}{3}} = 0.2^{-3} \cdot \left(\frac{1}{5}\right)^2 = \left(\frac{1}{5}\right)^{-3} \cdot \frac{1}{25} = 5^3 \cdot \frac{1}{25} = 125 \cdot \frac{1}{25} = 5;$$

$$4) \left(\frac{1}{9}\right)^{0.5} + 0.027^{-\frac{2}{3}} = \left(\frac{1}{3}\right)^1 + 0.3^{-2} = \frac{1}{3} + \left(\frac{10}{3}\right)^2 = \frac{1}{3} + \frac{100}{9} = 11\frac{4}{9}.$$

Zadatak 5. Izračunaj:

$$1) 16^{-0.25} + \left(\frac{1}{4}\right)^{-\frac{1}{2}}; \quad 2) 0.001^{-\frac{1}{3}} + 0.01^{-\frac{1}{2}};$$

$$3) 0.25^{-\frac{3}{2}} \cdot \left(\frac{1}{16}\right)^{-0.5}; \quad 4) \left(\frac{1}{8}\right)^{-\frac{2}{3}} \cdot (0.81)^{-0.5};$$

$$5) 16^{0.5} + \left(\frac{1}{16}\right)^{-0.75}; \quad 6) \left(\frac{27}{8}\right)^{-\frac{2}{3}} - 1.44^{-\frac{1}{2}}.$$

Rješenje.

$$1) 16^{-0.25} + \left(\frac{1}{4}\right)^{-\frac{1}{2}} = \left(\frac{1}{16}\right)^{\frac{1}{4}} + 4^{\frac{1}{2}} = \sqrt[4]{\frac{1}{16}} + \sqrt{4} = \frac{1}{2} + 2 = \frac{5}{2};$$

$$2) 0.001^{-\frac{1}{3}} + 0.01^{-\frac{1}{2}} = 10^{-3 \cdot (-\frac{1}{3})} + 10^{-2 \cdot (-\frac{1}{2})} = 10 + 10 = 20;$$

$$3) 0.25^{-\frac{3}{2}} \cdot \left(\frac{1}{16}\right)^{-0.5} = 0.5^{-3} \cdot \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^3 \cdot 2^2 = 2^3 \cdot 2^2 = 2^5 = 32;$$

$$4) \left(\frac{1}{8}\right)^{-\frac{2}{3}} \cdot (0.81)^{-0.5} = \left(\frac{1}{2}\right)^{-2} \cdot (0.9)^{-1} = 2^2 \cdot \frac{10}{9} = \frac{40}{9} = 4\frac{4}{9};$$

$$5) 16^{0.5} + \left(\frac{1}{16}\right)^{-0.75} = 2^2 + \left(\frac{1}{2}\right)^{-3} = 4 + 2^3 = 4 + 8 = 12;$$

$$6) \left(\frac{27}{8}\right)^{-\frac{2}{3}} - 1.44^{-\frac{1}{2}} = \left(\frac{3}{2}\right)^{-2} - 1.2^{-1} = \left(\frac{2}{3}\right)^2 - \frac{5}{6} = \frac{4}{9} - \frac{5}{6} = -\frac{7}{18}.$$

Zadatak 6. Potenciraj zadane izraze:

$$1) (x^{\frac{1}{2}} + y^{\frac{1}{2}})^2; \quad 2) (x^{\frac{1}{4}} - y^{\frac{1}{3}})^2; \quad 3) (a^{-\frac{1}{2}} + 2a)^2;$$

$$4) (a^{-1} + b^2)^2; \quad 5) (x^{-\frac{1}{2}} + x^{\frac{1}{2}})^2; \quad 6) (x + x^{\frac{1}{2}} + 1)^2;$$

$$7) (x^{-\frac{1}{2}} + 1 + x^{\frac{1}{2}})^2; \quad 8) (1 + x^{\frac{1}{3}} + x^{\frac{1}{2}})^2; \quad 9) (x^{\frac{1}{3}} - x^{-\frac{1}{3}})^3.$$

Rješenje.

$$1) x + 2x^{\frac{1}{2}}y^{\frac{1}{2}} + y; \quad 2) x^{\frac{1}{2}} - 2x^{\frac{1}{4}}y^{\frac{1}{3}} + y^{\frac{2}{3}}; \quad 3) a^{-1} + 4a^{\frac{1}{2}} + 4a^2;$$

$$4) a^{-2} + 2a^{-1}b^2 + b^4; \quad 5) x^{-1} + x + 2; \quad 6) x^2 + x + 1 + 2x^{3/2} + 2x + 2x^{\frac{1}{2}};$$

$$7) x^{-1} + x + 2x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + 3; \quad 8) 1 + x^{\frac{1}{2}} + x + 2x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + 2x^{3/4};$$

$$9) x - 3x^{\frac{1}{3}} + 3x^{-\frac{1}{3}} - x^{-1}.$$

Zadatak 7. Pojednostavni sljedeće algebarske izraze:

- 1) $\frac{x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{x^{-\frac{1}{2}} + x^{\frac{1}{2}}}$;
- 2) $\frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{(x^{\frac{1}{4}} - y^{\frac{1}{4}})^2}$;
- 3) $\frac{xy^{\frac{1}{2}} - x^{\frac{1}{2}}y}{x - y}$;
- 4) $\frac{x^{\frac{1}{4}}y^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{4}}}{x^{\frac{1}{2}} + 2x^{\frac{1}{4}}y^{\frac{1}{4}} + y^{\frac{1}{2}}}$.

Rješenje.

- 1) $\frac{x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{x^{-\frac{1}{2}} + x^{\frac{1}{2}}} = \frac{\frac{1}{\sqrt{x}} - \sqrt{x}}{\frac{1}{\sqrt{x}} + \sqrt{x}} = \frac{\frac{1-x}{\sqrt{x}}}{\frac{1+x}{\sqrt{x}}} = \frac{1-x}{1+x}$;
- 2) $\frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{(x^{\frac{1}{4}} - y^{\frac{1}{4}})^2} = \frac{(x^{\frac{1}{4}} - y^{\frac{1}{4}})(x^{\frac{1}{4}} + y^{\frac{1}{4}})}{(x^{\frac{1}{4}} - y^{\frac{1}{4}})^2} = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{4}} - y^{\frac{1}{4}}}$;
- 3) $\frac{xy^{\frac{1}{2}} - x^{\frac{1}{2}}y}{x - y} = \frac{x^{\frac{1}{2}}y^{\frac{1}{2}}(x^{\frac{1}{2}} - y^{\frac{1}{2}})}{(x^{\frac{1}{2}} - y^{\frac{1}{2}})(x^{\frac{1}{2}} + y^{\frac{1}{2}})} = \frac{x^{\frac{1}{2}}y^{\frac{1}{2}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}$;
- 4) $\frac{x^{\frac{1}{4}}y^{\frac{1}{2}} + x^{\frac{1}{2}}y^{\frac{1}{4}}}{x^{\frac{1}{2}} + 2x^{\frac{1}{4}}y^{\frac{1}{4}} + y^{\frac{1}{2}}} = \frac{x^{\frac{1}{4}}y^{\frac{1}{4}}(y^{\frac{1}{4}} + x^{\frac{1}{4}})}{(x^{\frac{1}{4}} + y^{\frac{1}{4}})^2} = \frac{x^{\frac{1}{4}}y^{\frac{1}{4}}}{x^{\frac{1}{4}} + y^{\frac{1}{4}}}$.

Zadatak 8. Izračunaj $(3^{-1} \cdot 2^{-2})^{-1}$ i $(3^{-1} + 2^{-2})^{-1}$.

Rješenje. $(3^{-1} \cdot 2^{-2})^{-1} = \left(\frac{1}{3} \cdot \frac{1}{4}\right)^{-1} = 12$; $(3^{-1} + 2^{-2})^{-1} = \left(\frac{1}{3} + \frac{1}{4}\right)^{-1} = \frac{12}{7}$.

Zadatak 9. Koliko je:

- 1) $(9^{-\frac{1}{2}} + (3\sqrt{3})^{-\frac{2}{3}}) \cdot (9^{-\frac{1}{2}} - (3\sqrt{3})^{-\frac{2}{3}})$;
- 2) $(16^{-0.25} - (2\sqrt{2})^{\frac{1}{3}})(16^{-0.25} + (2\sqrt{2})^{\frac{1}{3}})$;
- 3) $(\sqrt[4]{18} - 2^{-\frac{3}{4}})(\sqrt[4]{18} + 2^{-\frac{3}{4}})$;
- 4) $[(27^{-\frac{1}{4}})^{-\frac{2}{3}} - (\sqrt{125})^{\frac{1}{3}}] \cdot [(9^{\frac{1}{3}})^{0.75} - (0.04)^{-\frac{1}{4}}]$?

Rješenje.

- 1) $(9^{-\frac{1}{2}} + (3\sqrt{3})^{-\frac{2}{3}}) \cdot (9^{-\frac{1}{2}} - (3\sqrt{3})^{-\frac{2}{3}}) = (9^{-\frac{1}{2}})^2 - (3^{-\frac{2}{3}} \cdot 3^{\frac{2}{3}})^2$
 $= \frac{1}{9} - \frac{1}{9} = 0$;
- 2) $(16^{-0.25} - (2\sqrt{2})^{\frac{1}{3}})(16^{-0.25} + (2\sqrt{2})^{\frac{1}{3}}) = (2^{-1})^2 - (2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}})^2$
 $= \frac{1}{4} - 2 = -\frac{7}{4}$;

$$\begin{aligned}
 3) \quad & \left(\sqrt[4]{18} - 2^{-\frac{3}{4}}\right) \left(\sqrt[4]{18} + 2^{-\frac{3}{4}}\right) = \left(18^{\frac{1}{4}} - \left(\frac{1}{2}\right)^{\frac{3}{4}}\right) \left(18^{\frac{1}{4}} + \left(\frac{1}{2}\right)^{\frac{3}{4}}\right) \\
 & = 18^{\frac{1}{2}} - \left(\frac{1}{2}\right)^{\frac{3}{2}} = 3\sqrt{2} - \frac{1}{2\sqrt{2}} = \frac{11}{2\sqrt{2}}; \\
 4) \quad & \left[(27^{-\frac{1}{4}})^{-\frac{2}{3}} - (\sqrt{125})^{\frac{1}{3}}\right] \cdot \left[(9^{\frac{1}{3}})^{0.75} - (0.04)^{-\frac{1}{4}}\right] \\
 & = \left(3^{-\frac{3}{4} \cdot (-\frac{2}{3})} - 5^{\frac{2}{3} \cdot \frac{1}{3}}\right) \cdot \left(3^{\frac{2}{3} \cdot \frac{3}{4}} - \left(\frac{1}{25}\right)^{-\frac{1}{4}}\right) \\
 & = \left(3^{-\frac{3}{4} \cdot (-\frac{2}{3})} - 5^{\frac{2}{3} \cdot \frac{1}{3}}\right) \cdot \left(3^{\frac{2}{3} \cdot \frac{3}{4}} - \left(\frac{1}{25}\right)^{-\frac{1}{4}}\right) = \left(3^{\frac{1}{2}} - 5^{\frac{1}{3}}\right) \cdot \left(3^{\frac{1}{2}} - 5^{\frac{1}{3}}\right) \\
 & = 3 - 5 = -2.
 \end{aligned}$$

Zadatak 10. Ako je $a > 0$, izračunaj:

- | | |
|---|--|
| 1) $(a^{-\frac{1}{2}})^{-\frac{2}{3}} \cdot (a^{\frac{1}{3}})^{-4}$; | 2) $(a^2)^{-\frac{2}{3}} \cdot (a^{-\frac{1}{3}})^2$; |
| 3) $(a^{-\frac{3}{5}})^2 \cdot (a^{\frac{2}{5}})^3$; | 4) $(a^{-\frac{3}{4}} \cdot a^{\frac{3}{2}})^{-\frac{1}{3}} \cdot a \cdot \sqrt[4]{a}$; |
| 5) $(a^{-2} \cdot a^{\frac{1}{3}})^{\frac{2}{5}} \cdot \sqrt[4]{a^3}$; | 6) $(a^{-\frac{1}{2}} \cdot a^{\frac{2}{3}})^{-2} \cdot a \cdot \sqrt[3]{a}$; |
| 7) $(a^{-\frac{2}{3}} \cdot a^{\frac{3}{4}})^3 \cdot \sqrt[4]{a^3}$; | 8) $(a^{-\frac{3}{4}} : \sqrt[3]{a^4})^{-0.8} : (a\sqrt[3]{a})$. |

Rješenje.

$$\begin{aligned}
 1) \quad & (a^{-\frac{1}{2}})^{-\frac{2}{3}} \cdot (a^{\frac{1}{3}})^{-4} = a^{-\frac{1}{2} \cdot (-\frac{2}{3})} \cdot a^{\frac{1}{3} \cdot (-4)} = a^{\frac{1}{3}} \cdot a^{-\frac{4}{3}} = a^{\frac{1}{3} - \frac{4}{3}} = a^{-1} = \frac{1}{a}; \\
 2) \quad & (a^2)^{-\frac{2}{3}} \cdot (a^{-\frac{1}{3}})^2 = a^{2 \cdot (-\frac{2}{3})} \cdot a^{-2 \cdot \frac{1}{3}} = a^{-\frac{4}{3}} \cdot a^{-\frac{2}{3}} = a^{-\frac{4}{3} - \frac{2}{3}} = a^{-2} = \frac{1}{a^2}; \\
 3) \quad & (a^{-\frac{3}{5}})^2 \cdot (a^{\frac{2}{5}})^3 = a^{-2 \cdot \frac{3}{5}} \cdot a^{3 \cdot \frac{2}{5}} = a^{-\frac{6}{5}} \cdot a^{\frac{6}{5}} = a^{-\frac{6}{5} + \frac{6}{5}} = 1; \\
 4) \quad & (a^{-\frac{3}{4}} \cdot a^{\frac{3}{2}})^{-\frac{1}{3}} \cdot a \cdot \sqrt[4]{a} = (a^{-\frac{3}{4} + \frac{3}{2}})^{-\frac{1}{3}} \cdot a \cdot a^{\frac{1}{4}} = a^{\frac{3}{4} \cdot (-\frac{1}{3})} \cdot a^{\frac{5}{4}} = a^{-\frac{1}{4} + \frac{5}{4}} = a; \\
 5) \quad & (a^{-2} \cdot a^{\frac{1}{3}})^{\frac{2}{5}} \cdot \sqrt[4]{a^3} = (a^{-2 + \frac{1}{3}})^{\frac{2}{5}} \cdot a^{\frac{3}{4}} = a^{-\frac{5}{3} \cdot \frac{2}{5}} \cdot a^{\frac{3}{4}} = a^{-\frac{2}{3} + \frac{3}{4}} = a^{\frac{1}{12}}; \\
 6) \quad & (a^{-\frac{1}{2}} \cdot a^{\frac{2}{3}})^{-2} \cdot a \cdot \sqrt[3]{a} = (a^{-\frac{1}{2} + \frac{2}{3}})^{-2} \cdot a \cdot a^{\frac{1}{3}} = a^{\frac{1}{6} \cdot (-2)} \cdot a^{1 + \frac{1}{3}} = a^{-\frac{1}{3}} \cdot a^{\frac{4}{3}} \\
 & = a^{-\frac{1}{3} + \frac{4}{3}} = a; \\
 7) \quad & (a^{-\frac{2}{3}} \cdot a^{\frac{3}{4}})^3 \cdot \sqrt[4]{a^3} = (a^{-\frac{2}{3} + \frac{3}{4}})^3 \cdot a^{\frac{3}{4}} = a^{-\frac{1}{12} \cdot 3} \cdot a^{\frac{3}{4}} = a^{-\frac{1}{4} + \frac{3}{4}} = a; \\
 8) \quad & (a^{-\frac{3}{4}} : \sqrt[3]{a^4})^{-0.8} : (a\sqrt[3]{a}) = (a^{-\frac{3}{4}} : a^{\frac{4}{3}})^{-\frac{4}{5}} : (a \cdot a^{\frac{1}{3}}) \\
 & = (a^{-\frac{3}{4} - \frac{4}{3}})^{-\frac{4}{5}} : a^{1 + \frac{1}{3}} = a^{-\frac{25}{12} \cdot (-\frac{4}{5})} : a^{\frac{4}{3}} = a^{\frac{4}{3}} : a^{\frac{4}{3}} = a^{\frac{4}{3} - \frac{4}{3}} = a^0 = 1.
 \end{aligned}$$

Zadatak 11. Izračunaj:

$$1) \frac{10^{\frac{3}{5}} \cdot 2^{-0.6}}{5^{-1.4}}; \quad 2) \frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}}; \quad 3) \frac{12^{\frac{3}{4}} \cdot 2^{-0.5}}{3^{-\frac{1}{4}}}; \quad 4) \frac{12^{-\frac{1}{3}} \cdot 2^{\frac{2}{3}}}{6^{-\frac{1}{3}} \cdot 4^{\frac{2}{3}}}.$$

Rješenje.

$$\begin{aligned}
 1) \quad & \frac{10^{\frac{3}{5}} \cdot 2^{-0.6}}{5^{-1.4}} = 5^{\frac{3}{5}} \cdot 2^{\frac{3}{5}} \cdot 2^{-0.6} \cdot 5^{1.4} = 5^{\frac{3}{5}} \cdot 2^{\frac{3}{5}} \cdot 2^{-\frac{3}{5}} \cdot 5^{\frac{7}{5}} = 5^{\frac{3}{5} + \frac{7}{5}} \cdot 2^{\frac{3}{5} - \frac{3}{5}} \\
 & = 5^2 \cdot 2^0 = 25; \\
 2) \quad & \frac{15^{\frac{2}{3}} \cdot 3^{\frac{1}{3}}}{5^{-\frac{1}{3}}} = 5^{\frac{2}{3}} \cdot 3^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 5^{\frac{1}{3}} = 5^{\frac{2}{3} + \frac{1}{3}} \cdot 3^{\frac{2}{3} + \frac{1}{3}} = 5 \cdot 3 = 15; \\
 3) \quad & \frac{12^{\frac{3}{4}} \cdot 2^{-0.5}}{3^{-\frac{1}{4}}} = 3^{\frac{3}{4}} \cdot 4^{\frac{3}{4}} \cdot 2^{-\frac{1}{2}} \cdot 3^{\frac{1}{4}} = 3^{\frac{3}{4} + \frac{1}{4}} \cdot 2^{\frac{3}{2}} \cdot 2^{-\frac{1}{2}} = 3 \cdot 2^{\frac{3}{2} - \frac{1}{2}} = 3 \cdot 2 = 6;
 \end{aligned}$$

$$4) \frac{12^{-\frac{1}{3}} \cdot 2^{\frac{2}{3}}}{6^{-\frac{1}{3}} \cdot 4^{\frac{2}{3}}} = 3^{-\frac{1}{3}} \cdot 4^{-\frac{1}{3}} \cdot 2^{\frac{2}{3}} \cdot 6^{\frac{1}{3}} \cdot 4^{-\frac{2}{3}} = 3^{-\frac{1}{3}} \cdot 2^{-\frac{2}{3}} \cdot 2^{\frac{2}{3}} \cdot 3^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 2^{-\frac{2}{3}}$$

$$= 3^{-\frac{1}{3} + \frac{1}{3}} \cdot 2^{-\frac{2}{3} + \frac{2}{3} + \frac{1}{3} - \frac{2}{3}} = 1 \cdot 2^{-1} = \frac{1}{2}.$$

Zadatak 12. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^{-\frac{2}{3}} b^{\frac{1}{3}} \right)^{0.5} : \left(a^{-\frac{3}{4}} b^{-\frac{3}{4}} \right)^{\frac{2}{3}} \right]^{-\frac{3}{2}},$$

za $a = \frac{1}{16}$, $b = 2$.

Rješenje. $\left[\left(a^{-\frac{2}{3}} b^{\frac{1}{3}} \right)^{0.5} : \left(a^{-\frac{3}{4}} b^{-\frac{3}{4}} \right)^{\frac{2}{3}} \right]^{-\frac{3}{2}} = \left[\left(a^{-\frac{1}{3}} b^{\frac{1}{6}} \right) : \left(a^{-\frac{1}{2}} b^{-\frac{1}{2}} \right) \right]^{-\frac{3}{2}}$

$$= \left(a^{-\frac{1}{3}} b^{\frac{1}{6}} a^{\frac{1}{2}} b^{\frac{1}{2}} \right)^{-\frac{3}{2}} = \left(a^{-\frac{1}{3} + \frac{1}{2}} b^{\frac{1}{6} + \frac{1}{2}} \right)^{-\frac{3}{2}} = \left(a^{\frac{1}{6}} b^{\frac{2}{3}} \right)^{-\frac{3}{2}} = a^{-\frac{1}{4}} b^{-1}.$$

Uvrstimo $a = \frac{1}{16} = 2^{-4}$, $b = 2$:

$$(2^{-4})^{-\frac{1}{4}} 2^{-1} = 2 \cdot \frac{1}{2} = 1.$$

Zadatak 13. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} : \left(a^{-\frac{2}{3}} b \right)^2 \right]^{-\frac{1}{2}},$$

za $a = \frac{8}{27}$, $b = \frac{1}{81}$.

Rješenje. $\left[\left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} : \left(a^{-\frac{2}{3}} b \right)^2 \right]^{-\frac{1}{2}} = \left[\left(a^{-\frac{2}{3}} b^{\frac{1}{2}} \right) : \left(a^{-\frac{4}{3}} b^2 \right) \right]^{-\frac{1}{2}} = \left(a^{-\frac{2}{3}} b^{\frac{1}{2}} a^{\frac{4}{3}} b^{-2} \right)^{-\frac{1}{2}}$

$$= \left(a^{-\frac{2}{3} + \frac{4}{3}} b^{\frac{1}{2} - 2} \right)^{-\frac{1}{2}} = \left(a^{\frac{2}{3}} b^{-\frac{3}{2}} \right)^{-\frac{1}{2}} = a^{-\frac{1}{3}} b^{\frac{3}{4}}.$$

Uvrstimo $a = \frac{8}{27} = \left(\frac{2}{3}\right)^3$, $b = \frac{1}{81} = \left(\frac{1}{3}\right)^4$:

$$\left[\left(\frac{2}{3} \right)^3 \right]^{-\frac{1}{3}} \left[\left(\frac{1}{3} \right)^4 \right]^{\frac{3}{4}} = \left(\frac{2}{3} \right)^{-1} \left(\frac{1}{3} \right)^3 = \frac{3}{2} \cdot \frac{1}{27} = \frac{1}{18}.$$

Zadatak 14. Izračunaj vrijednost brojevnog izraza

$$\left[\left(a^{-\frac{2}{3}} b^{\frac{3}{4}} \right)^{-2} : \left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} \right]^{-\frac{1}{3}},$$

za $a = 8$, $b = \frac{1}{64}$.

Rješenje. $\left[\left(a^{-\frac{2}{3}} b^{\frac{3}{4}} \right)^{-2} : \left(a^2 b^{-1.5} \right)^{-\frac{1}{3}} \right]^{-\frac{1}{3}} = \left[\left(a^{\frac{4}{3}} b^{-\frac{3}{2}} \right) : \left(a^{-\frac{2}{3}} b^{\frac{1}{2}} \right) \right]^{-\frac{1}{3}}$

$$= \left(a^{\frac{4}{3}} b^{-\frac{3}{2}} a^{\frac{2}{3}} b^{-\frac{1}{2}} \right)^{-\frac{1}{3}} = \left(a^2 b^{-2} \right)^{-\frac{1}{3}} = \left(\frac{a}{b} \right)^{-\frac{2}{3}} = \left(\frac{b}{a} \right)^{\frac{2}{3}}.$$

$$\text{Uvrstimo } a = 8 = 2^3, b = \frac{1}{64} = \left(\frac{1}{2}\right)^6 = 2^{-6};$$

$$\left(\frac{2^{-6}}{2^3}\right)^{\frac{2}{3}} = (2^{-6} \cdot 2^{-3})^{\frac{2}{3}} = 2^{-9 \cdot \frac{2}{3}} = 2^{-6} = \frac{1}{64}.$$

Zadatak 15. Izračunaj:

$$\begin{array}{ll} 1) \frac{15^{-\frac{2}{3}} \cdot \sqrt[3]{1.8}}{0.04^{0.5}}; & 2) \frac{12^{-\frac{3}{4}} \cdot \sqrt[4]{27}}{\sqrt{0.005}}; \\ 3) \frac{72^{-\frac{1}{3}}}{0.25^{-\frac{3}{2}} \cdot \sqrt[3]{81}}; & 4) \frac{\sqrt[3]{0.125}}{48^{-\frac{3}{4}} \cdot \sqrt[4]{27}}. \end{array}$$

Rješenje.

$$1) \frac{15^{-\frac{2}{3}} \cdot \sqrt[3]{1.8}}{0.04^{0.5}} = \frac{3^{-\frac{2}{3}} \cdot 5^{-\frac{2}{3}} \cdot \left(\frac{9}{5}\right)^{\frac{1}{3}}}{0.2} = \frac{3^{-\frac{2}{3}} \cdot 5^{-\frac{2}{3}} \cdot 3^{\frac{2}{3}} \cdot 5^{-\frac{1}{3}}}{5^{-1}} = 3^0 \cdot 5^{-1} \cdot 5^1 = 5^0 = 1;$$

$$2) \frac{12^{-\frac{3}{4}} \cdot \sqrt[4]{27}}{\sqrt{0.005}} = \frac{3^{-\frac{3}{4}} \cdot 2^{-\frac{3}{4}} \cdot 3^{\frac{3}{4}}}{\left(\frac{1}{200}\right)^{\frac{1}{2}}} = 2^{-\frac{3}{2}} \cdot 200^{\frac{1}{2}} = 2^{-\frac{3}{2}} \cdot 10 \cdot 2^{\frac{1}{2}} = \frac{1}{2} \cdot 10 = 5;$$

$$3) \frac{72^{-\frac{1}{3}}}{0.25^{-\frac{3}{2}} \cdot \sqrt[3]{81}} = \frac{8^{-\frac{1}{3}} \cdot 9^{-\frac{1}{3}}}{\left(\frac{1}{4}\right)^{-\frac{3}{2}} \cdot 3^{\frac{4}{3}}} = \frac{2^{-1} \cdot 3^{-\frac{2}{3}}}{2^3 \cdot 3^{\frac{4}{3}}} = 2^{-1} \cdot 3^{-\frac{2}{3}} \cdot 2^{-3} \cdot 3^{-\frac{4}{3}} = 2^{-4} \cdot 3^{-2} = \frac{1}{16} \cdot \frac{1}{9} = \frac{1}{144};$$

$$4) \frac{\sqrt[3]{0.125}}{48^{-\frac{3}{4}} \cdot \sqrt[4]{27}} = \frac{\left(\frac{1}{8}\right)^{\frac{1}{3}}}{3^{-\frac{3}{4}} \cdot 16^{-\frac{3}{4}} \cdot 3^{\frac{3}{4}}} = \frac{2^{-1}}{2^{-3}} = 2^{-1} \cdot 2^3 = 4.$$

Zadatak 16. Jednostavan model koji povezuje trajanje oluje s površinom koju ona obuhvaća dan je formulom

$$T = 0.07 d^{\frac{3}{2}}.$$

Tu je d promjer (u miljama) područja pod olujom, a T vrijeme trajanja u satima.

Kolika je procjena trajanja oluje ako je obuhvaćeno područje promjera 200 km?

Rješenje.

Pretvorimo kilometre u milje. Vrijedi: 1 milja = 1.609 km pa je $200 \text{ km} = \frac{200}{1.609} = 124.3$ milje. Sada je

$$T = 0.07 \cdot 124.3^{\frac{3}{2}} \approx 0.07 \cdot 1385.83 = 97.$$

Oluja će trajati 97 sati.

Zadatak 17. Jednostavna formula za računanje ophodnog vremena planeta oko Sunca glasi:

$$T = 2 \times 10^{-10} d^{\frac{3}{2}}.$$

Ovdje je d udaljenost planeta od Sunca izražena u kilometrima, vrijeme T iskazuje se u danima.

Kolika je relativna pogreška ove formule? Zamijeni konstantu 2 onom koja će dati ispravnu vrijednost.

Saturnu je potrebno 29.46 godina za jedan ophod oko Sunca. Kolika je udaljenost Saturna od Sunca?

Rješenje. Uvrstimo podatke koji su poznati za Zemlju, $T = 365.25$ dana, $d = 149\,597\,870.7$ km, u danoj formuli umjesto konstante 2 pišemo x . Dobivamo $365.25 = x \cdot 10^{-10} \cdot 149\,597\,870.7^{\frac{3}{2}}$, slijedi da je $x \approx 1.9962$ pa je relativna pogreška $\frac{2 - 1.9962}{1.9962} = 0.0019$.

Prebacimo 29.46 godina u dane, to je $29.46 \cdot 365.25 = 10\,760.265$ dana. Uvrstimo u formulu: $10\,760.265 = 1.9962 \cdot 10^{-10} \cdot d^{\frac{3}{2}}$, slijedi $d \approx 1\,426\,962\,666$ km.

Zadatak 18. Opći oblik trećeg Keplerova zakona glasi

$$d = \left(\frac{GM}{4\pi^2} \right)^{\frac{1}{3}} T^{\frac{2}{3}}$$

Tu je d udaljenost (u metrima) satelita koji kruži oko tijela mase M , T vrijeme ophoda satelita (u sekundama) i $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ gravitacijska konstanta.

Znamo da je Mjesec udaljen od Zemlje prosječno 384 401 km, a vrijeme ophoda mu je 27 dana 7 sati 43 minute i 11.6 sekundi. Na temelju tih podataka izračunaj masu Zemlje.

Rješenje. Prvo zapišimo podatke u odgovarajućim mjernim jednicama: $d = 384\,401 \text{ km} = 384\,401\,000 \text{ m}$ i $T = 27 \text{ dana } 7 \text{ sati } 43 \text{ minute } 11.6 \text{ sekundi} = 2\,360\,591.6 \text{ sekundi}$. Iz dane formule izrazimo traženu nepoznanicu M , vrijedi $d^3 = \frac{GM}{4\pi^2} \cdot T^2$, odnosno $M = \frac{d^3 4\pi^2}{GT^2}$. Uvrstimo poznate podatke i dobivamo rezultat približno jednak $6.0332 \cdot 10^{24}$.

1.3. Računanje s korijenima

Zadatak 1. Provjeri i obrazloži:

$$1) \sqrt[4]{81} = 3; \quad 2) \sqrt[3]{0.125} = \frac{1}{2}; \quad 3) \sqrt[5]{\frac{1}{32}} = \frac{1}{2}; \quad 4) \sqrt[4]{\frac{16}{625}} = \frac{2}{5};$$

$$5) \sqrt[6]{64} = 2; \quad 6) \sqrt[3]{\frac{27}{125}} = \frac{3}{5}; \quad 7) \sqrt[3]{\frac{125}{216}} = \frac{5}{6}; \quad 8) \sqrt[4]{0.0625} = 0.5.$$

Rješenje.

$$1) \sqrt[4]{81} = \sqrt[4]{3^4} = 3; \quad 2) \sqrt[3]{0.125} = \sqrt[3]{\frac{1}{8}} = \sqrt[3]{\left(\frac{1}{2}\right)^3} = \frac{1}{2};$$

$$3) \sqrt[5]{\frac{1}{32}} = \sqrt[5]{\left(\frac{1}{2}\right)^5} = \frac{1}{2}; \quad 4) \sqrt[4]{\frac{16}{625}} = \sqrt[4]{\left(\frac{2}{5}\right)^4} = \frac{2}{5};$$

$$5) \sqrt[6]{64} = \sqrt[6]{2^6} = 2; \quad 6) \sqrt[3]{\frac{27}{125}} = \sqrt[3]{\left(\frac{3}{5}\right)^3} = \frac{3}{5};$$

$$7) \sqrt[3]{\frac{125}{216}} = \sqrt[3]{\left(\frac{5}{6}\right)^3} = \frac{5}{6}; \quad 8) \sqrt[4]{0.0625} = \sqrt[4]{0.5^4} = 0.5.$$

Zadatak 2. Izračunaj:

$$1) \sqrt{2.56 \cdot 10^{-2}} + 2 \cdot \sqrt[3]{0.008}; \quad 2) \sqrt[6]{9^3 \cdot 1000^2} - \sqrt{0.25 \cdot 10^2};$$

$$3) \sqrt[4]{1.6 \cdot 10^{-3}} + 2 \cdot \sqrt[3]{0.125}; \quad 4) \sqrt[3]{0.09 \cdot 300} + 5 \cdot \sqrt{0.16}.$$

Rješenje.

$$1) \sqrt{2.56 \cdot 10^{-2}} + 2 \cdot \sqrt[3]{0.008} = \sqrt{0.0256} + 2 \cdot \sqrt[3]{0.2^3} = \sqrt{0.16^2} + 2 \cdot 0.2$$

$$= 0.16 + 0.4 = 0.56;$$

$$2) \sqrt[6]{9^3 \cdot 1000^2} - \sqrt{0.25 \cdot 10^2} = \sqrt[6]{3^6 \cdot 10^6} - \sqrt{0.5^2 \cdot 10^2} = \sqrt[6]{30^6} - \sqrt{5^2}$$

$$= 30 - 5 = 25;$$

$$3) \sqrt[4]{1.6 \cdot 10^{-3}} + 2 \cdot \sqrt[3]{0.125} = \sqrt[4]{0.0016} + 2 \cdot \sqrt[3]{0.5^3} = \sqrt[4]{0.2^4} + 2 \cdot 0.5$$

$$= 0.2 + 1 = 1.2;$$

$$4) \sqrt[3]{0.09 \cdot 300} + 5 \cdot \sqrt{0.16} = \sqrt[3]{27} + 5 \cdot \sqrt{0.4^2} = \sqrt[3]{3^3} + 5 \cdot 0.4 = 3 + 2 = 5.$$

Zadatak 3. Izračunaj:

$$1) \sqrt[3]{8^4}; \quad 2) \sqrt[4]{4^6}; \quad 3) \sqrt[8]{16^2}; \quad 4) \sqrt[6]{27^2};$$

$$5) \sqrt[9]{125^6}; \quad 6) \sqrt[4]{81^3}; \quad 7) \sqrt[12]{64^8}; \quad 8) \sqrt[8]{625^4}.$$

Rješenje.

$$1) \sqrt[3]{8^4} = \sqrt[3]{2^{12}} = \sqrt[3]{(2^4)^3} = 2^4 = 16;$$

$$2) \sqrt[4]{4^6} = \sqrt[4]{2^{12}} = \sqrt[4]{(2^3)^4} = 2^3 = 8;$$

$$3) \sqrt[8]{16^2} = \sqrt[8]{2^8} = 2; \quad 4) \sqrt[6]{27^2} = \sqrt[6]{3^6} = 3;$$

$$5) \sqrt[9]{125^6} = \sqrt[9]{5^{18}} = \sqrt[9]{(5^2)^9} = 5^2 = 25;$$

$$6) \sqrt[4]{81^3} = \sqrt[4]{3^{12}} = \sqrt[4]{(3^3)^4} = 3^3 = 27;$$

$$7) \sqrt[12]{64^8} = \sqrt[12]{2^{48}} = \sqrt[12]{(2^4)^{12}} = 2^4 = 16;$$

$$8) \sqrt[8]{625^4} = \sqrt[8]{5^{16}} = \sqrt[8]{(5^2)^8} = 5^2 = 25.$$

Zadatak 4. Primjenjujući jednakost $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$, izračunaj:

$$1) \sqrt[3]{100} \cdot \sqrt[3]{10}; \quad 2) \sqrt[3]{12} \cdot \sqrt[3]{18}; \quad 3) \sqrt[4]{40} \cdot \sqrt[4]{250};$$

$$4) \sqrt[4]{50} \cdot \sqrt[4]{200}; \quad 5) \sqrt[5]{3^9} \cdot \sqrt[5]{3^{11}}; \quad 6) \sqrt[6]{4^3} \cdot \sqrt[6]{8^4}.$$

Rješenje.

$$1) \sqrt[3]{100} \cdot \sqrt[3]{10} = \sqrt[3]{100 \cdot 10} = \sqrt[3]{1000} = 10;$$

$$2) \sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 18} = \sqrt[3]{216} = 6;$$

$$3) \sqrt[4]{40} \cdot \sqrt[4]{250} = \sqrt[4]{40 \cdot 250} = \sqrt[4]{10000} = 10;$$

$$4) \sqrt[4]{50} \cdot \sqrt[4]{200} = \sqrt[4]{50 \cdot 200} = \sqrt[4]{10000} = 10;$$

$$5) \sqrt[5]{3^9} \cdot \sqrt[5]{3^{11}} = \sqrt[5]{3^9 \cdot 3^{11}} = \sqrt[5]{3^{20}} = \sqrt[5]{(3^4)^5} = 3^4 = 81;$$

$$6) \sqrt[6]{4^3} \cdot \sqrt[6]{8^4} = \sqrt[6]{2^6 \cdot 2^{12}} = \sqrt[6]{2^{18}} = \sqrt[6]{(2^3)^6} = 2^3 = 8.$$

Zadatak 5. Izračunaj:

$$1) \sqrt[3]{8 \cdot 27}; \quad 2) \sqrt[4]{625 \cdot 16}; \quad 3) \sqrt[3]{75 \cdot 45};$$

$$4) \sqrt[5]{3^{10} \cdot 0.5^{15}}; \quad 5) \sqrt[6]{\frac{1}{8^4} \cdot 9^3}; \quad 6) \sqrt[4]{25^6 \cdot \frac{1}{16^3}}.$$

Rješenje.

$$1) \sqrt[3]{8 \cdot 27} = \sqrt[3]{2^3 \cdot 3^3} = \sqrt[3]{6^3} = 6;$$

$$2) \sqrt[4]{625 \cdot 16} = \sqrt[4]{5^4 \cdot 2^4} = \sqrt[4]{10^4} = 10;$$

$$3) \sqrt[3]{75 \cdot 45} = \sqrt[3]{3 \cdot 25 \cdot 5 \cdot 9} = \sqrt[3]{3^3 \cdot 5^3} = \sqrt[3]{15^3} = 15;$$

$$4) \sqrt[5]{3^{10} \cdot 0.5^{15}} = \sqrt[5]{(3^2)^5 \cdot \left[\left(\frac{1}{2}\right)^3\right]^5} = \sqrt[5]{9^5 \cdot \left(\frac{1}{8}\right)^5} = \sqrt[5]{\left(\frac{9}{8}\right)^5} = \frac{9}{8};$$

$$5) \sqrt[6]{\frac{1}{8^4} \cdot 9^3} = \sqrt[6]{\left(\frac{1}{4}\right)^6 \cdot 3^6} = \sqrt[6]{\left(\frac{3}{4}\right)^6} = \frac{3}{4};$$

$$6) \sqrt[4]{25^6 \cdot \frac{1}{16^3}} = \sqrt[4]{5^{12} \cdot \left(\frac{1}{2}\right)^{12}} = \sqrt[4]{125^4 \cdot \left(\frac{1}{8}\right)^4} = \sqrt[4]{\left(\frac{125}{8}\right)^4} = \frac{125}{8}.$$

Zadatak 6. Izračunaj:

$$1) \frac{\sqrt[4]{3}}{\sqrt[4]{48}}; \quad 2) \frac{\sqrt[3]{40}}{\sqrt[3]{625}}; \quad 3) \frac{\sqrt[5]{486}}{\sqrt[5]{64}}; \quad 4) \frac{\sqrt[3]{1.25}}{\sqrt[3]{0.01}}.$$

Rješenje.

$$1) \frac{\sqrt[4]{3}}{\sqrt[4]{48}} = \sqrt[4]{\frac{3}{48}} = \sqrt[4]{\frac{1}{16}} = \sqrt[4]{\left(\frac{1}{2}\right)^4} = \frac{1}{2};$$

$$2) \frac{\sqrt[3]{40}}{\sqrt[3]{625}} = \sqrt[3]{\frac{40}{625}} = \sqrt[3]{\frac{8}{125}} = \sqrt[3]{\left(\frac{2}{5}\right)^3} = \frac{2}{5};$$

$$3) \frac{\sqrt[5]{486}}{\sqrt[5]{64}} = \sqrt[5]{\frac{486}{64}} = \sqrt[5]{\frac{243}{32}} = \sqrt[5]{\left(\frac{3}{2}\right)^5} = \frac{3}{2};$$

$$4) \frac{\sqrt[3]{1.25}}{\sqrt[3]{0.01}} = \sqrt[3]{\frac{1.25}{0.01}} = \sqrt[3]{125} = \sqrt[3]{5^3} = 5.$$

Zadatak 7. Provjeri jednakosti:

$$1) \sqrt[3]{7+5\sqrt{2}} = 1 + \sqrt{2}; \quad 2) \sqrt[4]{49+20\sqrt{6}} = \sqrt{2} + \sqrt{3};$$

$$3) \sqrt[3]{26-15\sqrt{3}} = 2 - \sqrt{3}; \quad 4) \sqrt[4]{28-16\sqrt{3}} = \sqrt{3} - 1.$$

Rješenje.

$$1) (1 + \sqrt{2})^3 = 1 + 3\sqrt{2} + 6 + 2\sqrt{2} = 7 + 5\sqrt{2};$$

$$2) (\sqrt{2} + \sqrt{3})^4 = \left[(\sqrt{2} + \sqrt{3})^2 \right]^2 = (5 + 2\sqrt{6})^2 = 25 + 20\sqrt{6} + 24 = 49 + 20\sqrt{6};$$

$$3) (2 - \sqrt{3})^3 = 8 - 12\sqrt{3} + 18 - 3\sqrt{3} = 26 - 15\sqrt{3};$$

$$4) (\sqrt{3} - 1)^4 = \left[(\sqrt{3} - 1)^2 \right]^2 = (3 - 2\sqrt{3} + 1)^2 = (4 - 2\sqrt{3})^2 = 16 - 16\sqrt{3} + 12 = 28 - 16\sqrt{3}.$$

Zadatak 8. Djelomice korjenuj:

$$1) \sqrt[3]{54}; \quad 2) \sqrt[3]{1080}; \quad 3) \sqrt[5]{160}; \quad 4) \sqrt[4]{80}; \quad 5) \sqrt[4]{324}.$$

Rješenje.

$$1) \sqrt[3]{54} = \sqrt[3]{2 \cdot 27} = \sqrt[3]{2 \cdot 3^3} = 3\sqrt[3]{2};$$

$$2) \sqrt[3]{1080} = \sqrt[3]{5 \cdot 8 \cdot 27} = 6\sqrt[3]{5};$$

$$3) \sqrt[5]{160} = \sqrt[5]{5 \cdot 32} = \sqrt[5]{5 \cdot 2^5} = 2\sqrt[5]{5};$$

$$4) \sqrt[4]{80} = \sqrt[4]{5 \cdot 16} = \sqrt[4]{5 \cdot 2^4} = 2\sqrt[4]{5};$$

$$5) \sqrt[4]{324} = \sqrt[4]{4 \cdot 81} = \sqrt[4]{4 \cdot 3^4} = 3\sqrt[4]{4}.$$

Zadatak 9. Pomnoži:

$$1) \sqrt{2} \cdot \sqrt[3]{2}; \quad 2) \sqrt{3} \cdot \sqrt[4]{27}; \quad 3) \sqrt[3]{4} \cdot \sqrt[4]{8};$$

$$4) \sqrt{3} \cdot \sqrt[3]{9} \cdot \sqrt[4]{27}; \quad 5) \sqrt[3]{25} \cdot \sqrt[4]{125} \cdot \sqrt[12]{5}; \quad 6) \sqrt[3]{\frac{1}{2}} \cdot \sqrt[6]{32} \cdot \sqrt[4]{8};$$

$$7) \sqrt[3]{4} \cdot \sqrt[4]{8} \cdot \sqrt[6]{32}; \quad 8) \sqrt[3]{4} \cdot \sqrt[4]{8} \cdot \sqrt[6]{\frac{1}{32}}.$$

Rješenje.

$$1) \sqrt{2} \cdot \sqrt[3]{2} = \sqrt[6]{2^3} \cdot \sqrt[6]{2^2} = \sqrt[6]{8 \cdot 4} = \sqrt[6]{32};$$

$$2) \sqrt{3} \cdot \sqrt[4]{27} = \sqrt[4]{3^2} \cdot \sqrt[4]{3^3} = \sqrt[4]{3^2 \cdot 3^3} = \sqrt[4]{3^5} = \sqrt[4]{3^4 \cdot 3} = 3\sqrt[4]{3};$$

$$3) \sqrt[3]{4} \cdot \sqrt[4]{8} = \sqrt[3]{2^2} \cdot \sqrt[4]{2^3} = \sqrt[12]{2^8} \cdot \sqrt[12]{2^9} = \sqrt[12]{2^8 \cdot 2^9} = \sqrt[12]{2^{17}} = \sqrt[12]{2^{12} \cdot 2^5} = 2\sqrt[12]{32};$$

$$4) \sqrt{3} \cdot \sqrt[3]{9} \cdot \sqrt[4]{27} = \sqrt{3} \cdot \sqrt[3]{3^2} \cdot \sqrt[4]{3^3} = \sqrt[12]{3^6} \cdot \sqrt[12]{3^8} \cdot \sqrt[12]{3^9} = \sqrt[12]{3^6 \cdot 3^8 \cdot 3^9} = \sqrt[12]{3^{23}} = \sqrt[12]{3^{12} \cdot 3^{11}} = 3\sqrt[12]{3^{11}};$$

$$5) \sqrt[3]{25} \cdot \sqrt[4]{125} \cdot \sqrt[12]{5} = \sqrt[3]{5^2} \cdot \sqrt[4]{5^3} \cdot \sqrt[12]{5} = \sqrt[12]{5^8} \cdot \sqrt[12]{5^9} \cdot \sqrt[12]{5} = \sqrt[12]{5^8 \cdot 5^9 \cdot 5} = \sqrt[12]{5^{18}} = \sqrt[12]{5^{12} \cdot 5^6} = 5\sqrt[12]{5^6} = 5\sqrt{5};$$

$$\begin{aligned}
 6) \quad & \sqrt[3]{\frac{1}{2}} \cdot \sqrt[6]{32} \cdot \sqrt[4]{8} = \sqrt[3]{2^{-1}} \cdot \sqrt[6]{2^5} \cdot \sqrt[4]{2^3} = \sqrt[12]{2^{-4}} \cdot \sqrt[12]{2^{10}} \cdot \sqrt[12]{2^9} \\
 & = \sqrt[12]{2^{-4} \cdot 2^{10} \cdot 2^9} = \sqrt[12]{2^{15}} = \sqrt[12]{2^{12} \cdot 2^3} = 2 \sqrt[12]{2^3} = 2 \sqrt[4]{2}; \\
 7) \quad & \sqrt[3]{4} \cdot \sqrt[4]{8} \cdot \sqrt[6]{32} = \sqrt[3]{2^2} \cdot \sqrt[4]{2^3} \cdot \sqrt[6]{2^5} = \sqrt[12]{2^8} \cdot \sqrt[12]{2^9} \cdot \sqrt[12]{2^{10}} = \sqrt[12]{2^8 \cdot 2^9 \cdot 2^{10}} \\
 & = \sqrt[12]{2^{27}} = \sqrt[12]{2^{24} \cdot 2^3} = \sqrt[12]{(2^2)^{12} \cdot 2^3} = 4 \sqrt[12]{2^3} = 4 \sqrt[4]{2}; \\
 8) \quad & \sqrt[3]{4} \cdot \sqrt[4]{8} \cdot \sqrt[6]{\frac{1}{32}} = \sqrt[3]{2^2} \cdot \sqrt[4]{2^3} \cdot \sqrt[6]{2^{-5}} = \sqrt[12]{2^8} \cdot \sqrt[12]{2^9} \cdot \sqrt[12]{2^{-10}} \\
 & = \sqrt[12]{2^8 \cdot 2^9 \cdot 2^{-10}} = \sqrt[12]{2^7}.
 \end{aligned}$$

Zadatak 10. Ako je $a > 0$, pomnoži:

- 1) $\sqrt{a} \cdot \sqrt[4]{a}$; 2) $\sqrt{a} \cdot \sqrt[3]{a^2}$; 3) $\sqrt[3]{a^2} \cdot \sqrt[4]{a^3}$;
 4) $\sqrt[3]{a} \cdot \sqrt[6]{a}$; 5) $\sqrt{a} \cdot \sqrt[3]{a} \cdot \sqrt[4]{a}$; 6) $\sqrt[3]{a} \cdot \sqrt[4]{a} \cdot \sqrt[6]{a}$;
 7) $\sqrt{a} \cdot \sqrt[4]{a^3} \cdot \sqrt[8]{a^7}$; 8) $a \cdot \sqrt[3]{a^2} \cdot \sqrt[6]{a^5}$.

Rješenje.

$$\begin{aligned}
 1) \quad & \sqrt{a} \cdot \sqrt[4]{a} = \sqrt[4]{a^2} \cdot \sqrt[4]{a} = \sqrt[4]{a^2 \cdot a} = \sqrt[4]{a^3}; \\
 2) \quad & \sqrt{a} \cdot \sqrt[3]{a^2} = \sqrt[6]{a^3} \cdot \sqrt[6]{a^4} = \sqrt[6]{a^3 \cdot a^4} = \sqrt[6]{a^7} = \sqrt[6]{a^6 \cdot a} = a \sqrt[6]{a}; \\
 3) \quad & \sqrt[3]{a^2} \cdot \sqrt[4]{a^3} = \sqrt[12]{a^8} \cdot \sqrt[12]{a^9} = \sqrt[12]{a^8 \cdot a^9} = \sqrt[12]{a^{17}} = \sqrt[12]{a^{12} \cdot a^5} = a \sqrt[12]{a^5}; \\
 4) \quad & \sqrt[3]{a} \cdot \sqrt[6]{a} = \sqrt[6]{a^2} \cdot \sqrt[6]{a} = \sqrt[6]{a^2 \cdot a} = \sqrt[6]{a^3} = \sqrt{a}; \\
 5) \quad & \sqrt{a} \cdot \sqrt[3]{a} \cdot \sqrt[4]{a} = \sqrt[12]{a^6} \cdot \sqrt[12]{a^4} \cdot \sqrt[12]{a^3} = \sqrt[12]{a^6 \cdot a^4 \cdot a^3} = \sqrt[12]{a^{13}} = \sqrt[12]{a^{12} \cdot a} \\
 & = a \sqrt[12]{a}; \\
 6) \quad & \sqrt[3]{a} \cdot \sqrt[4]{a} \cdot \sqrt[6]{a} = \sqrt[12]{a^4} \cdot \sqrt[12]{a^3} \cdot \sqrt[12]{a^2} = \sqrt[12]{a^4 \cdot a^3 \cdot a^2} = \sqrt[12]{a^9} \\
 & = \sqrt[4]{a^3}; \\
 7) \quad & \sqrt{a} \cdot \sqrt[4]{a^3} \cdot \sqrt[8]{a^7} = \sqrt[8]{a^4} \cdot \sqrt[8]{a^6} \cdot \sqrt[8]{a^7} = \sqrt[8]{a^4 \cdot a^6 \cdot a^7} = \sqrt[8]{a^{17}} = \sqrt[8]{a^{16} \cdot a} = a^2 \sqrt[8]{a}; \\
 8) \quad & a \cdot \sqrt[3]{a^2} \cdot \sqrt[6]{a^5} = \sqrt[6]{a^6} \cdot \sqrt[6]{a^4} \cdot \sqrt[6]{a^5} = \sqrt[6]{a^6 \cdot a^4 \cdot a^5} = \sqrt[6]{a^{15}} = \sqrt[6]{a^{12} \cdot a^3} = a^2 \sqrt[6]{a}.
 \end{aligned}$$

Zadatak 11. Pomnoži:

1) $\sqrt{2 + \sqrt{3}} \cdot \sqrt[4]{7 - 4\sqrt{3}}$; 2) $\sqrt[3]{1 + \sqrt{2}} \cdot \sqrt[6]{3 - 2\sqrt{2}}$.

Rješenje.

$$\begin{aligned}
 1) \quad & \sqrt{2 + \sqrt{3}} \cdot \sqrt[4]{7 - 4\sqrt{3}} = \sqrt[4]{(2 + \sqrt{3})^2} \cdot \sqrt[4]{7 - 4\sqrt{3}} = \sqrt[4]{(7 + 4\sqrt{3}) \cdot (7 - 4\sqrt{3})} \\
 & = \sqrt[4]{49 - 48} = \sqrt[4]{1} = 1; \\
 2) \quad & \sqrt[3]{1 + \sqrt{2}} \cdot \sqrt[6]{3 - 2\sqrt{2}} = \sqrt[6]{(1 + \sqrt{2})^2} \cdot \sqrt[6]{3 - 2\sqrt{2}} = \sqrt[6]{(3 + 2\sqrt{2}) \cdot (3 - 2\sqrt{2})} \\
 & = \sqrt[6]{9 - 8} = \sqrt[6]{1} = 1.
 \end{aligned}$$

Zadatak 12. Pojednostavi:

1) $\sqrt[4]{\sqrt[3]{a^4}} \cdot \sqrt[3]{\sqrt{a^3}}$; 2) $\sqrt[4]{\sqrt[3]{a^8}} : \sqrt[3]{\sqrt[3]{a^6}}$; 3) $\sqrt[5]{\sqrt[4]{a^{15}}} \cdot \sqrt[3]{\sqrt[4]{a^9}}$;
 4) $\sqrt[4]{\sqrt[3]{a^{10}}} \cdot \sqrt[3]{\sqrt{a^7}}$; 5) $\sqrt[3]{\sqrt[4]{a^9}} : \sqrt[3]{\sqrt[3]{a^4}}$; 6) $\sqrt[5]{\sqrt[3]{a^5}} \cdot \sqrt[4]{\sqrt[3]{a^2}}$;
 7) $\sqrt{\sqrt{a^6}} \cdot \sqrt[3]{\sqrt{a^9}}$; 8) $\sqrt[3]{\sqrt[4]{a^9}} : \sqrt[6]{\sqrt[3]{a^9}}$.

Rješenje.

$$\begin{aligned}
 1) \quad & \sqrt[4]{\sqrt[3]{a^4}} \cdot \sqrt[3]{\sqrt{a^3}} = \sqrt[3]{a} \cdot \sqrt{a} = \sqrt[6]{a^2} \cdot \sqrt[6]{a^3} = \sqrt[6]{a^5}; \\
 2) \quad & \sqrt[4]{\sqrt[3]{a^8}} : \sqrt[3]{\sqrt[3]{a^6}} = \sqrt[3]{a^2} : \sqrt[3]{a^2} = 1;
 \end{aligned}$$

- 3) $\sqrt[5]{\sqrt[4]{a^{15}}} \cdot \sqrt[3]{\sqrt[4]{a^9}} = \sqrt[4]{a^3} \cdot \sqrt[4]{a^3} = \sqrt[4]{a^6} = a\sqrt[4]{a^2} = a\sqrt{a}$;
- 4) $\sqrt[4]{\sqrt[3]{a^{10}}} \cdot \sqrt[3]{\sqrt{a^7}} = \sqrt{\sqrt[3]{a^5}} \cdot \sqrt[3]{\sqrt{a^7}} = \sqrt[6]{a^5} \cdot \sqrt[6]{a^7} = \sqrt[6]{a^{12}} = a^2$;
- 5) $\sqrt[3]{\sqrt[4]{a^9}} : \sqrt{\sqrt[3]{a^4}} = \sqrt[4]{a^3} : \sqrt[3]{a^2} = \sqrt[12]{a^9} : \sqrt[12]{a^8} = \sqrt[12]{a}$;
- 6) $\sqrt[5]{\sqrt[3]{a^5}} \cdot \sqrt[4]{\sqrt[3]{a^2}} = \sqrt[3]{a} \cdot \sqrt{\sqrt[3]{a}} = \sqrt[3]{a} \cdot \sqrt[6]{a} = \sqrt[6]{a^2} \cdot \sqrt[6]{a} = \sqrt[6]{a^3} = \sqrt{a}$;
- 7) $\sqrt{\sqrt{a^6}} \cdot \sqrt[3]{\sqrt{a^9}} = \sqrt{a^3} \cdot \sqrt{a^3} = \sqrt{a^6} = a^3$;
- 8) $\sqrt[3]{\sqrt[4]{a^9}} : \sqrt[6]{\sqrt[3]{a^9}} = \sqrt[4]{a^3} : \sqrt[6]{a^3} = \sqrt[12]{a^9} : \sqrt[12]{a^6} = \sqrt[12]{a^3} = \sqrt[4]{a}$.

Zadatak 13. Pojednostavni:

- 1) $\sqrt[3]{2\sqrt{2}}$; 2) $\sqrt{6 \cdot \sqrt[3]{6}}$; 3) $\sqrt[4]{3 \cdot \sqrt[3]{3}}$; 4) $\sqrt[4]{9 \cdot \sqrt[3]{9}}$;
- 5) $\sqrt[5]{4 \cdot \sqrt[4]{4}}$; 6) $\sqrt[3]{9 \cdot \sqrt[4]{3}}$; 7) $\sqrt[4]{25 \cdot \sqrt[3]{25}}$; 8) $\sqrt[7]{8 \cdot \sqrt[6]{8}}$.

Rješenje.

- 1) $\sqrt[3]{2\sqrt{2}} = \sqrt[3]{\sqrt{2^2} \cdot \sqrt{2}} = \sqrt[3]{\sqrt{2^3}} = \sqrt{2}$;
- 2) $\sqrt{6 \cdot \sqrt[3]{6}} = \sqrt{\sqrt[3]{6^3} \cdot \sqrt[3]{6}} = \sqrt{\sqrt[3]{6^4}} = \sqrt[3]{6^2} = \sqrt[3]{36}$;
- 3) $\sqrt[4]{3 \cdot \sqrt[3]{3}} = \sqrt[4]{\sqrt[3]{3^3} \cdot \sqrt[3]{3}} = \sqrt[4]{\sqrt[3]{3^4}} = \sqrt[3]{3}$;
- 4) $\sqrt[4]{9 \cdot \sqrt[3]{9}} = \sqrt[4]{3^2 \cdot \sqrt[3]{3^2}} = \sqrt[4]{\sqrt[3]{3^6} \cdot \sqrt[3]{3^2}} = \sqrt[4]{\sqrt[3]{3^8}} = \sqrt[3]{3^2} = \sqrt[3]{9}$;
- 5) $\sqrt[5]{4 \cdot \sqrt[4]{4}} = \sqrt[5]{\sqrt[4]{4^4} \cdot \sqrt[4]{4}} = \sqrt[5]{\sqrt[4]{4^5}} = \sqrt[4]{4} = \sqrt[4]{2^2} = \sqrt{2}$;
- 6) $\sqrt[3]{9 \cdot \sqrt[4]{3}} = \sqrt[3]{3^2 \cdot \sqrt[4]{3}} = \sqrt[3]{\sqrt[4]{3^8} \cdot \sqrt[4]{3}} = \sqrt[3]{\sqrt[4]{3^9}} = \sqrt[4]{3^3} = \sqrt[4]{27}$;
- 7) $\sqrt[4]{25 \cdot \sqrt[3]{25}} = \sqrt[4]{\sqrt[3]{25^3} \cdot \sqrt[3]{25}} = \sqrt[4]{\sqrt[3]{25^4}} = \sqrt[3]{25}$;
- 8) $\sqrt[7]{8 \cdot \sqrt[6]{8}} = \sqrt[7]{\sqrt[6]{8^6} \cdot \sqrt[6]{8}} = \sqrt[7]{\sqrt[6]{8^7}} = \sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt{2}$.

Zadatak 14. Izračunaj:

- 1) $\sqrt{4 \cdot \sqrt[3]{2}} \cdot \sqrt[3]{4 \cdot \sqrt{2}}$; 2) $\sqrt[3]{3 \cdot \sqrt[4]{27}} \cdot \sqrt[4]{3 \cdot \sqrt[3]{9}}$;
- 3) $\sqrt{5 \cdot \sqrt[3]{25}} \cdot \sqrt[3]{25 \cdot \sqrt[4]{125}}$; 4) $\sqrt[3]{2 \cdot \sqrt[4]{4}} \cdot \sqrt{8 \cdot \sqrt[3]{2}}$;
- 5) $\sqrt[3]{4 \cdot \sqrt{2}} : \sqrt{2 \cdot \sqrt[3]{4}}$; 6) $\sqrt[4]{8 \cdot \sqrt[3]{2}} : \sqrt{2 \cdot \sqrt[3]{4}}$;
- 7) $\sqrt[4]{27 \cdot \sqrt[3]{9}} : \sqrt[3]{9 \cdot \sqrt{3}}$; 8) $\sqrt[3]{25 \cdot \sqrt{5}} \cdot \sqrt[4]{125 \cdot \sqrt[3]{5}}$;
- 9) $\sqrt{\frac{1}{2} \cdot \sqrt[3]{4}} \cdot \sqrt[3]{\frac{1}{2} \cdot \sqrt[4]{8}}$; 10) $\sqrt[3]{9 \cdot \sqrt{\frac{1}{3}}} \cdot \sqrt{\frac{1}{3} \cdot \sqrt[3]{9}}$;
- 11) $\sqrt[3]{\frac{1}{4} \cdot \sqrt{2}} : \sqrt[6]{4 \cdot \sqrt{\frac{1}{4}}}$; 12) $\sqrt{2 \cdot \sqrt[3]{\frac{1}{4}}} : \sqrt[3]{\frac{1}{4} \cdot \sqrt{2}}$.

Rješenje.

- 1) $\sqrt{2^2 \cdot \sqrt[3]{2}} \cdot \sqrt[3]{2^2 \cdot \sqrt{2}} = \sqrt{\sqrt[3]{2^6} \cdot \sqrt[3]{2}} \cdot \sqrt[3]{\sqrt{2^4} \cdot \sqrt{2}} = \sqrt{\sqrt[3]{2^7}} \cdot \sqrt[3]{\sqrt{2^5}}$
 $= \sqrt[6]{2^7} \cdot \sqrt[6]{2^5} = \sqrt[6]{2^{12}} = 2^2 = 4$;
- 2) $\sqrt[3]{3 \cdot \sqrt[4]{3^3}} \cdot \sqrt[4]{3 \cdot \sqrt[3]{3^2}} = \sqrt[3]{\sqrt[4]{3^4} \cdot \sqrt[4]{3}} \cdot \sqrt[4]{\sqrt[3]{3^3} \cdot \sqrt[3]{3^2}} = \sqrt[3]{\sqrt[4]{3^7}} \cdot \sqrt[4]{\sqrt[3]{3^5}}$
 $= \sqrt[12]{3^7} \cdot \sqrt[12]{3^5} = \sqrt[12]{3^{12}} = 3$;

- 3) $\sqrt{5 \cdot \sqrt[3]{5^2}} \cdot \sqrt[3]{5^2 \cdot \sqrt[4]{5^3}} = \sqrt{\sqrt[3]{5^3} \cdot \sqrt[3]{5^2}} \cdot \sqrt[3]{\sqrt[4]{5^8} \cdot \sqrt[4]{5^3}} = \sqrt{\sqrt[3]{5^5}} \cdot \sqrt[3]{\sqrt[4]{5^{11}}}$
 $= \sqrt[6]{5^5} \cdot \sqrt[12]{5^{11}} = \sqrt[12]{5^{10}} \cdot \sqrt[12]{5^{11}} = \sqrt[12]{5^{21}} = 5 \sqrt[12]{5^9} = 5 \sqrt[4]{5^3} = 5 \sqrt[4]{125};$
- 4) $\sqrt[3]{2 \cdot \sqrt[3]{2^2}} \cdot \sqrt{2^3 \cdot \sqrt[3]{2}} = \sqrt[3]{\sqrt[3]{2^3} \cdot \sqrt[3]{2^2}} \cdot \sqrt{\sqrt[3]{2^9} \cdot \sqrt[3]{2}} = \sqrt[3]{\sqrt[3]{2^5}} \cdot \sqrt{\sqrt[3]{2^{10}}}$
 $= \sqrt[9]{2^5} \cdot \sqrt[6]{2^{10}} = \sqrt[18]{2^{10}} \cdot \sqrt[18]{2^{30}} = \sqrt[18]{2^{40}} = 4 \sqrt[18]{2^4} = 4 \sqrt[9]{4};$
- 5) $\sqrt[3]{2^2 \cdot \sqrt{2}} : \sqrt{2 \cdot \sqrt[3]{2^2}} = \sqrt[3]{\sqrt{2^4} \cdot \sqrt{2}} : \sqrt{\sqrt[3]{2^3} \cdot \sqrt[3]{2^2}} = \sqrt[3]{\sqrt{2^5}} : \sqrt{\sqrt[3]{2^5}}$
 $= \sqrt[6]{2^5} : \sqrt[6]{2^5} = 1;$
- 6) $\sqrt[4]{2^3 \cdot \sqrt[3]{2}} \cdot \sqrt{2 \cdot \sqrt[3]{2^2}} = \sqrt[4]{\sqrt[3]{2^9} \cdot \sqrt[3]{2}} \cdot \sqrt{\sqrt[3]{2^3} \cdot \sqrt[3]{2^2}} = \sqrt[4]{\sqrt[3]{2^{10}}} \cdot \sqrt{\sqrt[3]{2^5}}$
 $= \sqrt[12]{2^{10}} \cdot \sqrt[6]{2^5} = \sqrt[12]{2^{10}} \cdot \sqrt[12]{2^{10}} = 1;$
- 7) $\sqrt[4]{3^3 \cdot \sqrt[3]{3^2}} : \sqrt[3]{3^2 \cdot \sqrt{3}} = \sqrt[4]{\sqrt[3]{3^9} \cdot \sqrt[3]{3^2}} : \sqrt[3]{\sqrt{3^4} \cdot \sqrt{3}}$
 $= \sqrt[4]{\sqrt[3]{3^{11}}} : \sqrt[3]{\sqrt{3^5}} = \sqrt[12]{3^{11}} : \sqrt[6]{3^5} = \sqrt[12]{3^{11}} : \sqrt[12]{3^{10}} = \sqrt[12]{3};$
- 8) $\sqrt[3]{5^2 \cdot \sqrt{5}} \cdot \sqrt[4]{5^3 \cdot \sqrt[3]{5}} = \sqrt[3]{\sqrt{5^4} \cdot \sqrt{5}} \cdot \sqrt[4]{\sqrt[3]{5^9} \cdot \sqrt[3]{5}} = \sqrt[3]{\sqrt{5^5}} \cdot \sqrt[4]{\sqrt[3]{5^{10}}}$
 $= \sqrt[6]{5^5} \cdot \sqrt[12]{5^{10}} = \sqrt[12]{5^{10}} \cdot \sqrt[12]{5^{10}} = \sqrt[12]{5^{20}} = 5 \sqrt[12]{5^8} = 5 \sqrt[3]{5^2} = 5 \sqrt[3]{25};$
- 9) $\sqrt{2^{-1} \cdot \sqrt[3]{2^2}} \cdot \sqrt[3]{2^{-1} \cdot \sqrt[4]{2^3}} = \sqrt{\sqrt[3]{2^{-3}} \cdot \sqrt[3]{2^2}} \cdot \sqrt[3]{\sqrt[4]{2^{-4}} \cdot \sqrt[4]{2^3}} = \sqrt{\sqrt[3]{2^{-1}}} \cdot \sqrt[3]{\sqrt[4]{2^{-1}}}$
 $= \sqrt[6]{2^{-1}} \cdot \sqrt[12]{2^{-1}} = \sqrt[12]{2^{-2}} \cdot \sqrt[12]{2^{-1}} = \sqrt[12]{2^{-3}} = \sqrt[4]{2^{-1}} = \sqrt[4]{\frac{1}{2}} = \frac{1}{\sqrt[4]{2}};$
- 10) $\sqrt[3]{3^2 \cdot \sqrt{3^{-1}}} \cdot \sqrt{3^{-1} \cdot \sqrt[3]{3^2}} = \sqrt[3]{\sqrt{3^4} \cdot \sqrt{3^{-1}}} \cdot \sqrt{\sqrt[3]{3^{-3}} \cdot \sqrt[3]{3^2}}$
 $= \sqrt[3]{\sqrt{3^3}} \cdot \sqrt{\sqrt[3]{3^{-1}}} = \sqrt[6]{3^3} \cdot \sqrt[6]{3^{-1}} = \sqrt[6]{3^2} = \sqrt[3]{3};$
- 11) $\sqrt[3]{2^{-2} \cdot \sqrt{2}} : \sqrt[6]{2^2 \cdot \sqrt{2^{-2}}} = \sqrt[3]{\sqrt{2^{-4}} \cdot \sqrt{2}} : \sqrt[6]{\sqrt{2^4} \cdot \sqrt{2^{-2}}}$
 $= \sqrt[3]{\sqrt{2^{-3}}} : \sqrt[6]{\sqrt{2^2}} = \sqrt[6]{2^{-3}} : \sqrt[12]{2^2} = \sqrt[12]{2^{-6}} : \sqrt[12]{2^2} = \sqrt[12]{2^{-8}}$
 $= \sqrt[3]{2^{-2}} = \sqrt[3]{\frac{1}{4}} = \frac{1}{\sqrt[3]{4}};$
- 12) $\sqrt{2 \cdot \sqrt[3]{2^{-2}}} : \sqrt[3]{2^{-2} \cdot \sqrt{2}} = \sqrt{\sqrt[3]{2^3} \cdot \sqrt[3]{2^{-2}}} : \sqrt[3]{\sqrt{2^{-4}} \cdot \sqrt{2}}$
 $= \sqrt{\sqrt[3]{2}} : \sqrt[3]{\sqrt{2^{-3}}} = \sqrt[6]{2} : \sqrt[6]{2^{-3}} = \sqrt[6]{2^4} = \sqrt[3]{2^2} = \sqrt[3]{4}.$

Zadatak 15. Izračunaj:

- 1) $(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4})(\sqrt[3]{3} + \sqrt[3]{2});$
- 2) $(\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4})(\sqrt[3]{5} - \sqrt[3]{2});$
- 3) $(1 - \sqrt[3]{x})(1 + \sqrt[3]{x} + \sqrt[3]{x^2});$
- 4) $(\sqrt[3]{a} + \sqrt[6]{ab} + \sqrt[3]{b})(\sqrt[6]{a} - \sqrt[6]{b});$
- 5) $(\sqrt[3]{a^2} + \sqrt[3]{b^2})(\sqrt[3]{a^4} - \sqrt[3]{a^2b^2} + \sqrt[3]{b^4});$
- 6) $(a^2 - a\sqrt{b} + b)(a + \sqrt{b});$
- 7) $(a - \sqrt{a} \cdot \sqrt[4]{b} + \sqrt{b})(\sqrt{a} + \sqrt[4]{b});$
- 8) $\left(1 + \frac{1}{a} \sqrt[3]{ab^2} + \frac{1}{b} \sqrt[3]{a^2b}\right) \cdot \left(\sqrt[3]{\frac{a}{b}} - \sqrt[3]{\frac{b}{a}}\right).$

- Rješenje.**
- 1) $(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4})(\sqrt[3]{3} + \sqrt[3]{2}) = (\sqrt[3]{3})^3 + (\sqrt[3]{2})^3 = 3 + 2 = 5;$
 - 2) $(\sqrt[3]{25} + \sqrt[3]{10} + \sqrt[3]{4})(\sqrt[3]{5} - \sqrt[3]{2}) = (\sqrt[3]{5})^3 - (\sqrt[3]{2})^3 = 5 - 2 = 3;$

$$3) (1 - \sqrt[3]{x})(1 + \sqrt[3]{x} + \sqrt[3]{x^2}) = (1 - \sqrt[3]{x}) [1 + \sqrt[3]{x} + (\sqrt[3]{x})^2] = 1^3 - (\sqrt[3]{x})^3 = 1 - x;$$

$$4) (\sqrt[3]{a} + \sqrt[6]{ab} + \sqrt[3]{b})(\sqrt[6]{a} - \sqrt[6]{b}) = (\sqrt[6]{a^2} + \sqrt[6]{ab} + \sqrt[6]{b^2})(\sqrt[6]{a} - \sqrt[6]{b}) \\ = (\sqrt[6]{a})^3 - (\sqrt[6]{b})^3 = \sqrt{a} - \sqrt{b};$$

$$5) (\sqrt[3]{a^2} + \sqrt[3]{b^2})(\sqrt[3]{a^4} - \sqrt[3]{a^2b^2} + \sqrt[3]{b^4}) = (\sqrt[3]{a^2})^3 + (\sqrt[3]{b^2})^3 = a^2 + b^2;$$

$$6) (a^2 - a\sqrt{b} + b)(a + \sqrt{b}) = (a^2 - a\sqrt{b} + (\sqrt{b})^2)(a + \sqrt{b}) = a^3 + (\sqrt{b})^3 \\ = a^3 + b\sqrt{b};$$

$$7) (a - \sqrt{a} \cdot \sqrt[4]{b} + \sqrt{b})(\sqrt{a} + \sqrt[4]{b}) = ((\sqrt{a})^2 - \sqrt{a} \cdot \sqrt[4]{b} + (\sqrt[4]{b})^2)(\sqrt{a} + \sqrt[4]{b}) \\ = (\sqrt{a})^3 + (\sqrt[4]{b})^3 = a\sqrt{a} + \sqrt[4]{b^3};$$

$$8) \left(1 + \frac{1}{a} \sqrt[3]{ab^2} + \frac{1}{b} \sqrt[3]{a^2b}\right) \cdot \left(\sqrt[3]{\frac{a}{b}} - \sqrt[3]{\frac{b}{a}}\right) \\ = \left(\sqrt[3]{\frac{b^2}{s^2}} + 1 + \sqrt[3]{\frac{a^2}{b^2}}\right) \cdot \left(\sqrt[3]{\frac{a}{b}} - \sqrt[3]{\frac{b}{a}}\right) \\ = \left(\sqrt[3]{\frac{b^2}{s^2}} + \sqrt[3]{\frac{a}{b}} \cdot \sqrt[3]{\frac{b}{a}} + \sqrt[3]{\frac{a^2}{b^2}}\right) \cdot \left(\sqrt[3]{\frac{a}{b}} - \sqrt[3]{\frac{b}{a}}\right) \\ = \left(\sqrt[3]{\frac{a}{b}}\right)^3 - \left(\sqrt[3]{\frac{b}{a}}\right)^3 = \frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{ab}.$$

Zadatak 16. Izračunaj:

$$1) (\sqrt{a} + \sqrt[3]{b})(a - \sqrt[6]{a^3b^2} + \sqrt[3]{b^2}) \text{ za } a = \frac{1}{4}, b = 0.125;$$

$$2) (\sqrt[3]{a} - 2\sqrt[6]{b})(\sqrt[3]{a^2} + 2\sqrt[6]{a^2b} + 4\sqrt[3]{b}) \text{ za } a = 2, b = 4;$$

$$3) (\sqrt[3]{a^2} - 2\sqrt{b})(a \cdot \sqrt[3]{a} + 2\sqrt[6]{a^4b^3} + 4b), \text{ za } a = -\frac{1}{2\sqrt{2}}, b = \sqrt[3]{\frac{1}{4}};$$

$$4) (\sqrt[6]{a} + b\sqrt{b})(\sqrt[3]{a} - b\sqrt[6]{ab^3} + b^3) \text{ za } a = b = 0.25.$$

Rješenje.

1) Prvo sredimo izraz:

$$(\sqrt{a} + \sqrt[3]{b})(a - \sqrt[6]{a^3b^2} + \sqrt[3]{b^2}) = (\sqrt{a} + \sqrt[3]{b}) \left((\sqrt{a})^2 - \sqrt{a}\sqrt[3]{b} + (\sqrt[3]{b})^2 \right) \\ = (\sqrt{a})^3 + (\sqrt[3]{b})^3 = \sqrt{a^3} + b.$$

Uvrstimo sada vrijednosti za a i b :

$$\sqrt{a^3} + b = \sqrt{\left(\frac{1}{4}\right)^3} + 0.125 = \sqrt{\left(\frac{1}{2}\right)^6} + \frac{1}{8} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}.$$

2) Prvo sredimo izraz:

$$(\sqrt[3]{a} - 2\sqrt[6]{b})(\sqrt[3]{a^2} + \sqrt[3]{a} \cdot 2\sqrt[6]{b} + 2^2\sqrt[6]{b^2}) = (\sqrt[3]{a})^3 - (2\sqrt[6]{b})^3 = a - 8\sqrt{b}.$$

Uvrstimo sada vrijednosti za a i b :

$$a - 8\sqrt{b} = 2 - 8\sqrt{4} = 2 - 16 = -14.$$

3) Prvo sredimo izraz:

$$\begin{aligned} (\sqrt[3]{a^2} - 2\sqrt{b})(a \cdot \sqrt[3]{a} + 2\sqrt[6]{a^4b^3} + 4b) &= (\sqrt[3]{a^2} - 2\sqrt{b}) \left(\sqrt[3]{a^4} + \sqrt[3]{a^2} \cdot 2\sqrt{b} + (2b)^2 \right) \\ &= (\sqrt[3]{a^2})^3 - (2\sqrt{b})^3 = a^2 - 8\sqrt{b^3}. \end{aligned}$$

Uvrstimo sada vrijednosti za a i b :

$$\left(-\frac{1}{2\sqrt{2}}\right)^2 - 8\sqrt{\left(\sqrt[3]{\frac{1}{4}}\right)^3} = \frac{1}{8} - 8\sqrt{\frac{1}{4}} = \frac{1}{8} - 8 \cdot \frac{1}{2} = -\frac{31}{8}.$$

4) Prvo sredimo izraz:

$$\begin{aligned} (\sqrt[6]{a} + b\sqrt{b})(\sqrt[3]{a} - b\sqrt[6]{ab^3} + b^3) &= (\sqrt[6]{a} + b\sqrt{b})(\sqrt[6]{a^2} - \sqrt[6]{a} \cdot b\sqrt{b} + (b\sqrt{b})^2) \\ &= (\sqrt[6]{a})^3 + (b\sqrt{b})^3 = \sqrt{a} + b^3\sqrt{b^3} = \sqrt{a} + b^4\sqrt{b}. \end{aligned}$$

Uvrstimo sada vrijednosti za a i b ($a = b = 0.25 = \frac{1}{4}$):

$$\sqrt{a} + b^4\sqrt{b} = \frac{1}{2} + \frac{1}{256}\sqrt{\frac{1}{4}} = \frac{1}{2} + \frac{1}{256} \cdot \frac{1}{2} = \frac{257}{512}.$$

Zadatak 17. Ako su a i b pozitivni brojevi, pojednostavni:

$$1) \frac{\sqrt[3]{a^2b} + \sqrt[3]{ab^2}}{a\sqrt[3]{b} - b\sqrt[3]{a}}; \quad 2) \frac{(\sqrt[3]{a^2} - \sqrt[3]{b^2})(\sqrt[3]{a^2} + \sqrt[3]{b^2})}{a^2 \cdot \sqrt[3]{b^2} - b^2 \cdot \sqrt[3]{a^2}}.$$

Rješenje.

$$\begin{aligned} 1) \frac{\sqrt[3]{a^2b} + \sqrt[3]{ab^2}}{a\sqrt[3]{b} - b\sqrt[3]{a}} &= \frac{\sqrt[3]{ab}(\sqrt[3]{a} + \sqrt[3]{b})}{\sqrt[3]{a^3b} - \sqrt[3]{ab^3}} = \frac{\sqrt[3]{ab}(\sqrt[3]{a} + \sqrt[3]{b})}{\sqrt[3]{ab}(\sqrt[3]{a^2} - \sqrt[3]{b^2})} \\ &= \frac{(\sqrt[3]{a} + \sqrt[3]{b})}{(\sqrt[3]{a} - \sqrt[3]{b})(\sqrt[3]{a} + \sqrt[3]{b})} = \frac{1}{\sqrt[3]{a} - \sqrt[3]{b}}; \\ 2) \frac{(\sqrt[3]{a^2} - \sqrt[3]{b^2})(\sqrt[3]{a^2} + \sqrt[3]{b^2})}{a^2 \cdot \sqrt[3]{b^2} - b^2 \cdot \sqrt[3]{a^2}} &= \frac{\sqrt[3]{a^4} - \sqrt[3]{b^4}}{\sqrt[3]{a^6b^2} - \sqrt[3]{a^2b^6}} = \frac{\sqrt[3]{a^4} - \sqrt[3]{b^4}}{\sqrt[3]{a^2b^2}(\sqrt[3]{a^4} - \sqrt[3]{b^4})} \\ &= \frac{1}{\sqrt[3]{a^2b^2}}. \end{aligned}$$

Zadatak 18. Popravi pogreške u sljedećim izrazima:

$$\begin{aligned} 1) x^{\frac{4}{3}} &= \sqrt[4]{x^5}; & 2) \sqrt[3]{80} &= \sqrt[3]{16} \sqrt[3]{5} = 2\sqrt[3]{5}; \\ 3) \sqrt[3]{\sqrt[4]{a}} &= \sqrt[7]{a}; & 4) x^{\frac{2}{3}} \cdot x^{\frac{1}{2}} &= x^{\frac{2}{3} + \frac{1}{2}} = x^{\frac{7}{6}} = \sqrt[6]{x^7}. \end{aligned}$$

Rješenje.

$$\begin{aligned} 1) x^{\frac{4}{3}} &= \sqrt[3]{x^4}; & 2) \sqrt[3]{80} &= \sqrt[3]{16} \cdot \sqrt[3]{5} = \sqrt[3]{2^3 \cdot 2} \sqrt[3]{5} = 2\sqrt[3]{10}; \\ 3) \sqrt[3]{\sqrt[4]{a}} &= \sqrt[12]{a}; & 4) x^{\frac{2}{3}} \cdot x^{\frac{1}{2}} &= x^{\frac{2}{3} + \frac{1}{2}} = x^{\frac{7}{6}} = \sqrt[6]{x^7}. \end{aligned}$$

Zadatak 19. Racionaliziraj nazivnik u razlomku:

$$1) \frac{1}{\sqrt{2} \cdot \sqrt[3]{2}}; \quad 2) \frac{10}{\sqrt{5} \cdot \sqrt{5}}; \quad 3) \frac{2}{\sqrt[4]{2} \cdot \sqrt[3]{2}}; \quad 4) \frac{1}{\sqrt[3]{4} \sqrt[4]{2}}$$

Rješenje.

$$1) \frac{1}{\sqrt{2} \cdot \sqrt[3]{2}} = \frac{1}{\sqrt{\sqrt[3]{2^3} \cdot \sqrt[3]{2}}} = \frac{1}{\sqrt{\sqrt[3]{2^4}}} = \frac{1}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}}{2};$$

$$2) \frac{10}{\sqrt{5} \cdot \sqrt{5}} = \frac{10}{\sqrt{5^2 \cdot 5}} = \frac{10}{\sqrt{5^3}} = \frac{10}{\sqrt[4]{5^3}} \cdot \frac{\sqrt[4]{5}}{\sqrt[4]{5}} = \frac{10 \sqrt[4]{5}}{5} = 2 \sqrt[4]{5};$$

$$3) \frac{2}{\sqrt[4]{2} \cdot \sqrt[3]{2}} = \frac{2}{\sqrt[4]{\sqrt[3]{2^3} \cdot \sqrt[3]{2}}} = \frac{2}{\sqrt[4]{\sqrt[3]{2^4}}} = \frac{2}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{2 \sqrt[3]{2^2}}{2} = \sqrt[3]{4};$$

$$4) \frac{1}{\sqrt[3]{4} \sqrt[4]{2}} = \frac{1}{\sqrt[3]{2^2} \sqrt[4]{2}} = \frac{1}{\sqrt[3]{4} \sqrt[4]{2^9}} = \frac{1}{\sqrt[4]{2^3}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \frac{\sqrt[4]{2}}{2}.$$

Zadatak 20. Racionaliziraj nazivnik u razlomku:

$$1) \frac{1}{\sqrt{2} - \sqrt[4]{3}}; \quad 2) \frac{1}{\sqrt[4]{5} + \sqrt[4]{3}}; \quad 3) \frac{1}{\sqrt[3]{3} - 1};$$

$$4) \frac{1}{\sqrt[3]{2} + \sqrt[6]{2}}; \quad 5) \frac{2}{\sqrt[3]{9} + \sqrt[3]{15} + \sqrt[3]{25}}; \quad 6) \frac{3}{1 - \sqrt[3]{2} + \sqrt[3]{4}}.$$

Rješenje.

$$1) \frac{1}{\sqrt{2} - \sqrt[4]{3}} = \frac{1}{\sqrt[4]{4} - \sqrt[4]{3}} \cdot \frac{\sqrt[4]{4} + \sqrt[4]{3}}{\sqrt[4]{4} + \sqrt[4]{3}} = \frac{\sqrt[4]{4} + \sqrt[4]{3}}{\sqrt{4} - \sqrt{3}} \cdot \frac{\sqrt{4} + \sqrt{3}}{\sqrt{4} + \sqrt{3}}$$

$$= \frac{(\sqrt[4]{4} + \sqrt[4]{3})(\sqrt{4} + \sqrt{3})}{4 - 3} = (\sqrt{2} + \sqrt[4]{3})(2 + \sqrt{3});$$

$$2) \frac{1}{\sqrt[4]{5} + \sqrt[4]{3}} \cdot \frac{\sqrt[4]{5} - \sqrt[4]{3}}{\sqrt[4]{5} - \sqrt[4]{3}} = \frac{\sqrt[4]{5} - \sqrt[4]{3}}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{(\sqrt[4]{5} - \sqrt[4]{3})(\sqrt{5} + \sqrt{3})}{5 - 3}$$

$$= \frac{1}{2} (\sqrt[4]{5} - \sqrt[4]{3})(\sqrt{5} + \sqrt{3}) = \frac{1}{2} (\sqrt[4]{5} - \sqrt[4]{3})(\sqrt{5} + \sqrt{3});$$

$$3) \frac{1}{\sqrt[3]{3} - 1} \cdot \frac{\sqrt[3]{3^2} + \sqrt[3]{3} + 1}{\sqrt[3]{3^2} + \sqrt[3]{3} + 1} = \frac{\sqrt[3]{3^2} + \sqrt[3]{3} + 1}{(\sqrt[3]{3})^3 - 1^3} = \frac{\sqrt[3]{9} + \sqrt[3]{3} + 1}{3 - 1}$$

$$= \frac{1}{2} (\sqrt[3]{9} + \sqrt[3]{3} + 1);$$

$$4) \frac{1}{\sqrt[3]{2} + \sqrt[6]{2}} \cdot \frac{\sqrt[3]{2^2} - \sqrt[3]{2} \sqrt[6]{2} + \sqrt[6]{2^2}}{\sqrt[3]{2^2} - \sqrt[3]{2} \sqrt[6]{2} + \sqrt[6]{2^2}} = \frac{\sqrt[3]{4} - \sqrt[6]{2^2} \sqrt[6]{2} + \sqrt[6]{2}}{(\sqrt[3]{2})^3 + (\sqrt[6]{2})^3}$$

$$= \frac{\sqrt[3]{4} - \sqrt{2} + \sqrt[3]{2}}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 + \sqrt{2}} = \frac{(\sqrt[3]{4} - \sqrt{2} + \sqrt[3]{2})(2 - \sqrt{2})}{4 - 2}$$

$$= \frac{1}{2} (\sqrt[3]{4} - \sqrt{2} + \sqrt[3]{2})(2 - \sqrt{2});$$

$$\begin{aligned}
 5) \quad \frac{2}{\sqrt[3]{9} + \sqrt[3]{15} + \sqrt[3]{25}} &= \frac{2}{(\sqrt[3]{3})^2 + \sqrt[3]{3 \cdot 5} + (\sqrt[3]{5})^2} \cdot \frac{\sqrt[3]{3} - \sqrt[3]{5}}{\sqrt[3]{3} - \sqrt[3]{5}} \\
 &= \frac{2 \cdot (\sqrt[3]{3} - \sqrt[3]{5})}{(\sqrt[3]{3})^3 - (\sqrt[3]{5})^3} = \frac{2 \cdot (\sqrt[3]{3} - \sqrt[3]{5})}{3 - 5} = -(\sqrt[3]{3} - \sqrt[3]{5}) = \sqrt[3]{5} - \sqrt[3]{3}; \\
 6) \quad \frac{3}{1 - \sqrt[3]{2} + \sqrt[3]{4}} &= \frac{3}{1 - \sqrt[3]{2} + (\sqrt[3]{2})^2} \cdot \frac{1 + \sqrt[3]{2}}{1 + \sqrt[3]{2}} = \frac{3 \cdot (1 + \sqrt[3]{2})}{1^3 + (\sqrt[3]{2})^3} \\
 &= \frac{3 \cdot (1 + \sqrt[3]{2})}{1 + 2} = 1 + \sqrt[3]{2}.
 \end{aligned}$$

Zadatak 21. Oplošje O kocke čiji je brid duljine a je $6a^2$. Obujam V iste kocke je a^3 .

- 1) Izrazi obujam kocke kao funkciju njezina oplošja.
- 2) Ako je zadan obujam kocke, koliko je njezino oplošje?

Rješenje.

- 1) Vrijedi: $V^2 = a^6$, $O^3 = 6^3 a^6$, $V^2 = \frac{O^3}{6^3} \implies V = \sqrt{\frac{O^3}{6^3}}$.
- 2) $O^3 = 6^3 \cdot V^2$, $O = 6\sqrt[3]{V^2}$.

1.4. Grafovi potencija i korijena

Zadatak 1. Odredi prirodnu domenu funkcije:

- 1) $\sqrt[4]{2-x}$;
- 2) $\sqrt[6]{2-x^2}$;
- 3) $\sqrt[3]{x^2-2}$;
- 4) $\sqrt[5]{\frac{x-1}{x+2}}$.

Rješenje.

- 1) $2-x \geq 0 \implies x \leq 2$, $\mathcal{D} = \langle -\infty, 2 \rangle$.
- 2) $2-x^2 \geq 0 \implies x^2 \leq 2 \implies -\sqrt{2} \leq x \leq \sqrt{2}$, $\mathcal{D} = [-\sqrt{2}, \sqrt{2}]$.
- 3) Treći korijen definiran je za sve realne brojeve pa je $\mathcal{D} = \mathbb{R}$.
- 4) Peti korijen definiran je za sve realne brojeve, jedini uvjet u ovom slučaju je $x+2 \neq 0 \implies x \neq -2$ pa je $\mathcal{D} = \mathbb{R} \setminus \{-2\}$.

Zadatak 2. Odredi prirodnu domenu funkcije:

- 1) $\frac{\sqrt{2-x}}{\sqrt[3]{x-1}}$;
- 2) $\frac{\sqrt[6]{x+3}}{\sqrt[5]{5-x}}$;
- 3) $\sqrt{\sqrt[3]{x}-2}$;
- 4) $\sqrt{3-x^4}$.

Rješenje.

- 1) $2-x \geq 0 \implies x \leq 2$ te $x-1 \neq 0 \implies x \neq 1$, $\mathcal{D} = \langle -\infty, 1 \rangle \cup \langle 1, 2 \rangle$.
- 2) $x+3 \geq 0 \implies x \geq -3$ te $5-x \neq 0 \implies x \neq 5$, $\mathcal{D} = [-3, 5) \cup \langle 5, \infty \rangle$.

$$3) \sqrt[3]{x} - 2 \geq 0 \implies \sqrt[3]{x} \geq 2 \implies x \geq 8, \mathcal{D} = [8, \infty).$$

$$4) 3 - x^4 \geq 0 \implies x^4 \leq 3 \implies -\sqrt[4]{3} \leq x \leq \sqrt[4]{3}, \mathcal{D} = [-\sqrt[4]{3}, \sqrt[4]{3}].$$

Zadatak 3. Nacrtaj graf funkcije:

$$1) f_1(x) = \sqrt[3]{x} - 2;$$

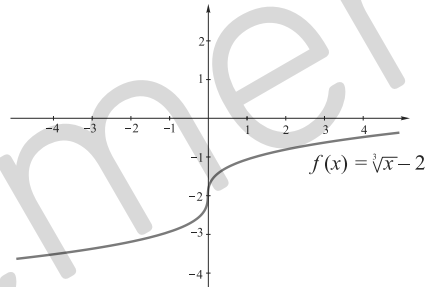
$$2) f_2(x) = \sqrt[3]{-x};$$

$$3) f_3(x) = \sqrt[3]{2-x};$$

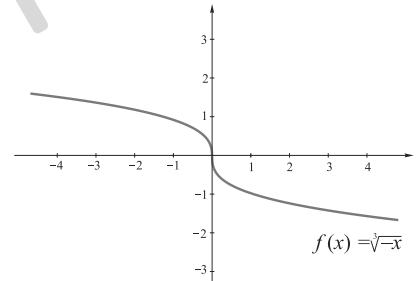
$$4) f_4(x) = 2 - \sqrt[3]{x-2}.$$

Rješenje.

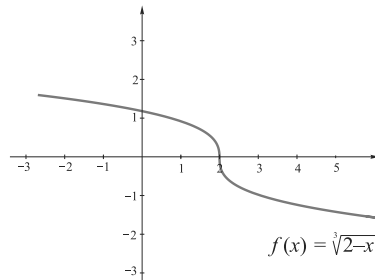
1)



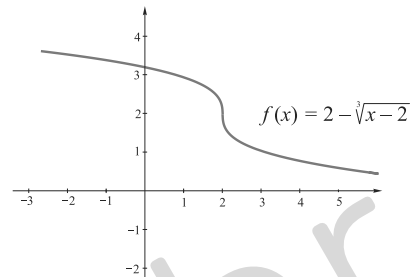
2)



3)



4)



Zadatak 4. Ispitaj koja se od točaka nalazi iznad, a koja ispod grafa funkcije $f(x) =$

$$\frac{\sqrt[6]{x} + 1}{1 - x^{-\frac{2}{3}}}$$

$$1) (0.05, -0.26);$$

$$2) (0.96, -71.23);$$

$$3) (1.15, 22.12);$$

$$4) (12.04, 4.12).$$

Rješenje.

$$1) \text{ ispod, jer je } f(0.05) = -0.252347 > -0.26;$$

$$2) \text{ iznad, jer je } f(0.96) = -72.2486 < -71.23;$$

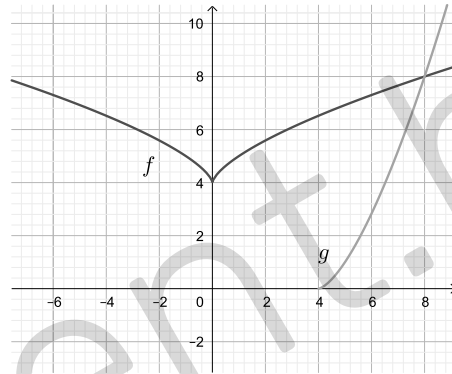
$$3) \text{ ispod, jer je } f(1.15) = 22.7455 > 22.12;$$

$$4) \text{ iznad, jer je } f(12.04) = 3.105 < 4.12.$$

Zadatak 5. U kojoj se točki sijeku grafovi funkcija $f(x) = x^{\frac{2}{3}} + 4$ i $g(x) = (x-4)^{\frac{3}{2}}$?

Rješenje.

Rješenje se nazire sa slike. Grafovi ovih funkcija sijeku se samo u jednoj točki. To je točka $(8, 8)$ koja leži na oba grafa.



Zadatak 6. Za velike vrijednosti varijable x koja funkcija poprima veće vrijednosti?

- 1) $f(x) = x^8$ ili $g(x) = 100x^7$;
- 2) $f(x) = x$ ili $g(x) = x^{\frac{3}{4}} + 100$;
- 3) $f(x) = x$ ili $g(x) = 100x^{\frac{3}{4}}$;
- 4) $f(x) = x^{\frac{1}{2}}$ ili $g(x) = 10x^{\frac{1}{3}}$.

Rješenje. Za velike vrijednosti varijable x veće vrijednosti će poprimiti funkcija koja sadrži potenciju s većim eksponentom. U svakom od zadataka to je funkcija $f(x)$.