



# 6 Trigonometrija trokuta

## 6.1. Trigonometrijske funkcije kutova u trokutu

**Zadatak 1.** Dva kuta,  $\alpha$  i  $\beta$ ,  $0^\circ < \alpha$ ,  $\beta < 90^\circ$  komplementarna su ako je  $\alpha + \beta = 90^\circ$ . Odredi komplement kuta  $\alpha$  ako je:

- 1)  $\alpha = 38^\circ$ ;    2)  $\alpha = 47^\circ 15'$ ;  
3)  $\alpha = 82^\circ 49' 33''$ ;    4)  $\alpha = 11^\circ 11' 11''$ .

*Rješenje.*

1) Iz  $\alpha + \beta = 90^\circ$  slijedi da je  $\beta = 90^\circ - \alpha = 90^\circ - 38^\circ = 52^\circ$ ;  
2)  $1^\circ = 60'$ , pa  $90^\circ$  možemo zapisati kao  $89^\circ$  i  $60'$ . Slijedi:  $\beta = 90^\circ - \alpha = 89^\circ 60' - 47^\circ 15' = 42^\circ 45'$ ;  
3)  $1^\circ = 60'$ , a  $1' = 60''$  pa  $90^\circ$  možemo zapisati kao  $89^\circ 59' 60''$ . Sada je  $\beta = 90^\circ - \alpha = 89^\circ 59' 60'' - 82^\circ 49' 33'' = 7^\circ 10' 27''$ ;  
4)  $\beta = 90^\circ - \alpha = 89^\circ 59' 60'' - 11^\circ 11' 11'' = 78^\circ 48' 49''$ .

**Zadatak 2.** Dva kuta,  $\alpha$  i  $\beta$ ,  $0^\circ < \alpha$ ,  $\beta < 180^\circ$  suplementarna su ako je  $\alpha + \beta = 180^\circ$ . Odredi suplement kuta  $\alpha$  ako je:

- 1)  $\alpha = 33^\circ$ ;    2)  $\alpha = 111^\circ 11' 11''$ ;  
3)  $\alpha = 79^\circ 59' 59''$ ;    4)  $\alpha = 100^\circ 01' 01''$ .

*Rješenje.*

1)  $\alpha + \beta = 180^\circ$ ,  $\alpha = 33^\circ \Rightarrow \beta = 180^\circ - 30^\circ = 147^\circ$ ;  
2)  $\alpha + \beta = 180^\circ$ ,  $\alpha = 111^\circ 11' 11'' \Rightarrow \beta = 180^\circ - 111^\circ 11' 11'' = 179^\circ 59' 60'' - 111^\circ 11' 11'' = 68^\circ 48' 49''$ ;  
3)  $\alpha + \beta = 180^\circ$ ,  $\alpha = 79^\circ 59' 59'' \Rightarrow \beta = 180^\circ - 79^\circ 59' 59'' = 179^\circ 59' 60'' - 79^\circ 59' 59'' = 100^\circ 0' 1''$ ;  
4)  $\alpha + \beta = 180^\circ$ ,  $\alpha = 100^\circ 01' 01'' \Rightarrow \beta = 180^\circ - 100^\circ 01' 01'' = 179^\circ 59' 60'' - 100^\circ 01' 01'' = 79^\circ 58' 59''$ .

**Zadatak 3.** Mjeru kuta  $\alpha$  zadanu u stupnjevima izrazi u stupnjevima, minutama i sekundama:

- 1)  $\alpha = 13.715^\circ$ ;    2)  $\alpha = -122.4445$ ;  
3)  $\alpha = 133.2345^\circ$ ;    4)  $\alpha = -47.6534^\circ$ .

*Rješenje.*

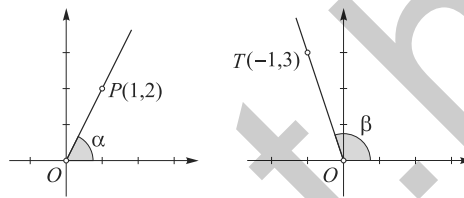
1)  $\alpha = 13.715^\circ = 13^\circ + 0.715 \cdot 60' = 13^\circ 42.9' = 13^\circ 42' + 0.9 \cdot 60'' = 13^\circ 42' 54''$ ;  
2)  $\alpha = -122.4445^\circ = -122^\circ - 0.4445 \cdot 60' = -122^\circ 26' - 0.67 \cdot 60'' = -122^\circ 26' 40''$ ;  
3)  $\alpha = 133.2345^\circ = 133^\circ + 0.2345 \cdot 60' = 133^\circ 14' + 0.07 \cdot 60'' = 133^\circ 14' 4''$ ;  
4)  $\alpha = -47.6534^\circ = -47^\circ - 0.6534 \cdot 60' = -47^\circ 39' - 0.204 \cdot 60'' = -47^\circ 39' 12''$ .

**Zadatak 4.** Mjeru kuta izrazi u stupnjevima:

- 1)  $45^\circ 15' 33''$ ;    2)  $95^\circ 27' 18''$ ;  
3)  $75^\circ 24' 48''$ ;    4)  $101^\circ 11' 10''$ .

- Rješenje.*
- 1)  $45^{\circ}15'33'' = 45^{\circ}15' + 33 : 60' = 45^{\circ}15.55' = 45^{\circ} + 15.55 : 60^{\circ} = 45.2592^{\circ};$
  - 2)  $95^{\circ}27'18'' = 95^{\circ}27' + 18 : 60' = 95^{\circ}27.3' = 95^{\circ} + 27.3 : 60^{\circ} = 95.455^{\circ};$
  - 3)  $75^{\circ}24'48'' = 75^{\circ}24' + 48 : 60' = 75^{\circ}24.8' = 75^{\circ} + 24.8 : 60^{\circ} = 75.413^{\circ};$
  - 4)  $101^{\circ}11'10'' = 101^{\circ}11' + 10 : 60' = 101^{\circ}11.1666667' = 101^{\circ} + 11.1\dot{6} : 60^{\circ} = 101.186\dot{1}^{\circ}.$

**Zadatak 5.** Odredi četiri trigonometrijske funkcije kutova zadanih podacima sa slike.

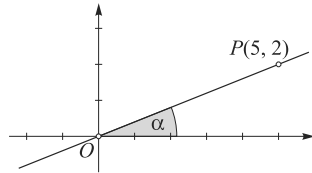


- Rješenje.*
- Za kut  $\alpha$ :  $r = \sqrt{1^2 + 2^2} = \sqrt{5},$   
 $\sin \alpha = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5},$   
 $\cos \alpha = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5},$   
 $\operatorname{tg} \alpha = \frac{2}{1} = 2,$   
 $\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha} = \frac{1}{2}.$
- Za kut  $\beta$ :  $r = \sqrt{(-1)^2 + 3^2} = \sqrt{10},$   
 $\sin \beta = \frac{y}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10},$   
 $\cos \beta = \frac{x}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10},$   
 $\operatorname{tg} \beta = -\operatorname{tg}(180^{\circ} - \beta) = -\frac{3}{1} = -3,$   
 $\operatorname{ctg} \beta = \frac{1}{\operatorname{tg} \beta} = -\frac{1}{3}.$

**Zadatak 6.** Jedan krak kuta  $\alpha$  je na osi  $Ox$ , vrh u ishodištu, a drugi krak na pravcu. Odredi sinus, kosinus i tangens kuta  $\alpha$ .

- 1)  $2x - 5y = 0;$
- 2)  $x - 3y = 0;$
- 3)  $2x + 3y = 0;$
- 4)  $x + 2y = 0.$

Rješenje. 1)



$$r = \sqrt{x^2 + y^2} = \sqrt{5^2 + 2^2} = \sqrt{29},$$

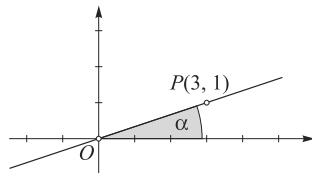
$$\sin \alpha = \frac{y}{r} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29},$$

$$\cos \alpha = \frac{x}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29},$$

$$\operatorname{tg} \alpha = \frac{y}{x} = \frac{2}{5},$$

$$\alpha = \cos^{-1} \left( \frac{5\sqrt{29}}{29} \right) = 21^\circ 48' 5''.$$

2)



$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 1^2} = \sqrt{10},$$

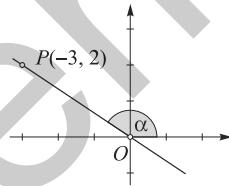
$$\sin \alpha = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10},$$

$$\cos \alpha = \frac{x}{r} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10},$$

$$\operatorname{tg} \alpha = \frac{y}{x} = \frac{1}{3},$$

$$\alpha = \cos^{-1} \left( \frac{3\sqrt{10}}{10} \right) = 18^\circ 26' 6''.$$

3)



$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{13},$$

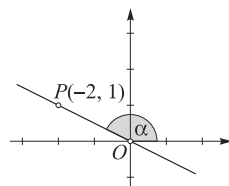
$$\sin \alpha = \frac{y}{r} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13},$$

$$\cos \alpha = \frac{x}{r} = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13},$$

$$\operatorname{tg} \alpha = \frac{y}{x} = \frac{2}{-3} = -\frac{2}{3},$$

$$\alpha = \cos^{-1} \left( -\frac{3\sqrt{13}}{13} \right) = 146^\circ 18' 36''.$$

4)



$$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + 1^2} = \sqrt{5},$$

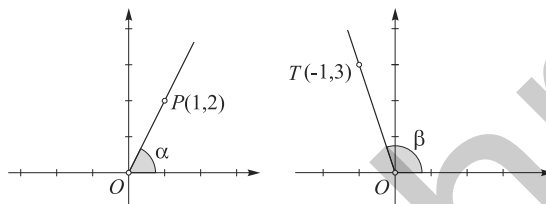
$$\sin \alpha = \frac{y}{r} = \frac{1}{\sqrt{5}} = \frac{1\sqrt{5}}{5},$$

$$\cos \alpha = \frac{x}{r} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5},$$

$$\operatorname{tg} \alpha = \frac{y}{x} = \frac{1}{-2} = -\frac{1}{2},$$

$$\alpha = \cos^{-1} \left( -\frac{2\sqrt{5}}{5} \right) = 153^{\circ}26'6''.$$

**Zadatak 7.** Odredi kutove zadane podatcima sa slika.



*Rješenje.* Kut  $\alpha$  je šiljasti pa će računalo iz vrijednosti  $\operatorname{tg} \alpha = \frac{2}{1} = 2$  dati njegovu vrijednost  $\alpha = 63^{\circ}26'06''$ .

Kut  $\beta$  je tup. Neka je  $\gamma$  njegov suplement, tj.  $\gamma = 180^{\circ} - \beta$ . Kut  $\gamma$  je šiljasti pa će računalo iz vrijednosti  $\operatorname{tg} \gamma = \frac{3}{1} = 3$  dati njegovu vrijednost  $\gamma = 71^{\circ}33'54''$ . Sad možemo odrediti kut  $\beta$ :  $\gamma = 180^{\circ} - \beta \implies \beta = 180^{\circ} - \gamma \implies \beta = 180^{\circ} - 71^{\circ}33'54'' \implies \beta = 108^{\circ}26'06''$ .

**Zadatak 8.** Za zadane kutove izračunaj vrijednosti trigonometrijskih funkcija. Rezultate upiši u bilježnicu, s četiri decimale.

kut	sin	cos	tg	ctg
25°				
50°				
75°				
100°				
125°				
150°				
175°				

*Rješenje.*

kut	sin	cos	tg	ctg
25°	0.4226	0.9063	0.4663	2.1445
50°	0.7660	0.6428	1.1918	0.8391
75°	0.9659	0.2588	3.7321	0.2679
100°	0.9848	-0.1736	-5.6713	-0.1763
125°	0.8192	-0.5736	-1.4281	-0.7002
150°	0.5	-0.8660	-0.5774	-1.7321
175°	0.0872	-0.9962	-0.0875	-11.43

**Zadatak 9.** Za zadane kutove izračunaj vrijednosti trigonometrijskih funkcija. Rezultate upiši u bilježnicu, s potrebnim brojem decimala.

kut	sin	cos	tg	ctg
15°26'				
42°18'				
66°13'12''				
98°18'				
122°5'				
171°26'3''				

*Rješenje.*

kut	sin	cos	tg	ctg
15°26'	0.2661	0.9639	0.2761	3.6222
42°10'	0.6730	0.396	0.9099	1.0990
66°13'12''	0.915100	0.403226	2.26945	0.440636
98°18'	0.9895	-0.1444	-6.855	-0.1459
122°5'	0.8473	-0.5312	-1.595	-0.6269
171°26'3''	0.148946	-0.988845	-0.150626	-6.63897

**Zadatak 10.** Odredi kut  $\alpha$  ako je:

- 1)  $\sin \alpha = 0.3616$ ;    2)  $\sin \alpha = 0.251378$ ;    3)  $\cos \alpha = 0.313$ ;  
 4)  $\cos \alpha = -0.1433$ ;    5)  $\operatorname{tg} \alpha = 0.725$ ;    6)  $\operatorname{tg} \alpha = 3.21277$ ;  
 7)  $\operatorname{ctg} \alpha = 0.1414$ ;    8)  $\operatorname{ctg} \alpha = -0.0588$ .

*Rješenje.*

- 1)  $\alpha = 21^\circ 12'$  ili  $\alpha = 158^\circ 48'$   
 2)  $\alpha = 14^\circ 33' 33''$  ili  $\alpha = 165^\circ 26' 27''$ ;  
 3)  $\alpha = 71^\circ 46''$ ;    4)  $\alpha = 98^\circ 14'$ ;    5)  $\alpha = 35^\circ 57'$ ;    6)  $\alpha = 72^\circ 43' 39''$ ;  
 7)  $\alpha = 81^\circ 57'$ ;    8)  $\alpha = 93^\circ 22'$ .

**Zadatak 11.** Iz zadane vrijednosti trigonometrijske funkcije odredi mjeru suplementarnog kuta. Rezultat zapiši u stupnjevima.

- 1)  $\cos \alpha = 0.3$ ;    2)  $\cos \beta = -0.8$ ;  
 3)  $\operatorname{tg} \gamma = 0.5$ ;    4)  $\sin \delta = 0.2$ .

*Rješenje.*

- 1)  $\alpha = 107.4576^\circ$ ;    2)  $\beta = 36.8699^\circ$ ;    3)  $\gamma = 153.4349$ ;  
 4)  $\delta = 168.4630$  ili  $\delta = 11.5370$ .

## 6.2. Poučak o sinusima

**Zadatak 1.** Izračunaj duljine ostalih dviju stranica i treći kut trokuta ako je:

- 1)  $a = 21 \text{ cm}$ ,  $\alpha = 66^\circ$ ,  $\beta = 52^\circ$ ;
- 2)  $a = 7.3 \text{ cm}$ ,  $\beta = 86^\circ$ ,  $\gamma = 51^\circ$ ;
- 3)  $b = 13.2 \text{ cm}$ ,  $\alpha = 21^\circ 48'$ ,  $\beta = 123^\circ 42'$ ;
- 4)  $b = 44.5 \text{ cm}$ ,  $\alpha = 103^\circ 28'$ ,  $\gamma = 41^\circ 33'$ ;
- 5)  $c = 10 \text{ cm}$ ,  $\alpha = 88^\circ$ ,  $\gamma = 12^\circ$ ;
- 6)  $c = 0.89 \text{ cm}$ ,  $\alpha = 28^\circ$ ,  $\beta = 34^\circ$ .

*Rješenje.*

- 1)  $a = 21 \text{ cm}$   
 $\alpha = 66^\circ$   
 $\beta = 52^\circ$

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$b, c, \gamma = ?$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 66^\circ - 52^\circ = 62^\circ$$

$$b = \frac{a \sin \beta}{\sin \alpha} = \frac{21 \text{ cm} \cdot \sin 52^\circ}{\sin 66^\circ} = 18.11 \text{ cm}$$

$$c = \frac{a \sin \gamma}{\sin \alpha} = \frac{21 \text{ cm} \cdot \sin 62^\circ}{\sin 66^\circ} = 20.3 \text{ cm}$$

- 2)  $a = 7.3 \text{ cm}$   
 $\beta = 86^\circ$   
 $\gamma = 51^\circ$

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$b, c, \alpha = ?$

$$\alpha = 180^\circ - \beta - \gamma = 180^\circ - 86^\circ - 51^\circ = 43^\circ$$

$$b = \frac{a \sin \beta}{\sin \alpha} = \frac{7.3 \text{ cm} \cdot \sin 86^\circ}{\sin 43^\circ} = 10.68 \text{ cm}$$

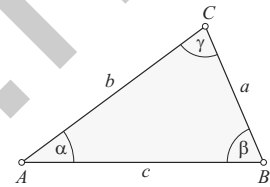
$$c = \frac{a \sin \gamma}{\sin \alpha} = \frac{7.3 \text{ cm} \cdot \sin 51^\circ}{\sin 43^\circ} = 8.32 \text{ cm}$$

- 3)  $b = 13.2 \text{ cm}$   
 $\alpha = 21^\circ 48'$   
 $\beta = 123^\circ 42'$

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$a, c, \gamma = ?$

$$\begin{aligned} \gamma &= 180^\circ - \alpha - \beta = 180^\circ - 21^\circ 48' - 123^\circ 42' = 180^\circ - 144^\circ 90' \\ &= 180^\circ - 145^\circ 30' = 179^\circ 60' - 145^\circ 30' = 34^\circ 30' \end{aligned}$$



$$a = \frac{b \sin \alpha}{\sin \beta} = \frac{13.2 \text{ cm} \cdot \sin 21^\circ 48'}{\sin 123^\circ 42'} = 5.89 \text{ cm}$$

$$c = \frac{a \sin \gamma}{\sin \alpha} = \frac{5.89 \text{ cm} \cdot \sin 21^\circ 48'}{\sin 34^\circ 30'} = 8.99 \text{ cm}$$

4)  $b = 44.5 \text{ cm}$

$$\alpha = 103^\circ 28'$$

$$\gamma = 41^\circ 33'$$

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$$a, c, \beta = ?$$

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 103^\circ 28' - 41^\circ 33' = 34^\circ 59'$$

$$a = \frac{b \sin \alpha}{\sin \beta} = \frac{44.5 \text{ cm} \cdot \sin 103^\circ 28'}{\sin 34^\circ 49'} = 75.48 \text{ cm}$$

$$c = \frac{a \sin \gamma}{\sin \alpha} = \frac{75.48 \text{ cm} \cdot \sin 41^\circ 33'}{\sin 103^\circ 28'} = 51.48 \text{ cm}$$

5)  $c = 10 \text{ cm}$

$$\alpha = 88^\circ$$

$$\gamma = 12^\circ$$

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$$a, b, \beta = ?$$

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 88^\circ - 12^\circ = 80^\circ$$

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{10 \text{ cm} \cdot \sin 88^\circ}{\sin 12^\circ} = 48.07 \text{ cm}$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{10 \text{ cm} \cdot \sin 80^\circ}{\sin 12^\circ} = 47.37 \text{ cm}$$

6)  $c = 0.89 \text{ cm}$

$$\alpha = 28^\circ$$

$$\beta = 34^\circ$$

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$$a, b, \gamma = ?$$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 28^\circ - 34^\circ = 118^\circ$$

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{0.89 \text{ cm} \cdot \sin 28^\circ}{\sin 118^\circ} = 0.47 \text{ cm}$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{0.89 \text{ cm} \cdot \sin 34^\circ}{\sin 118^\circ} = 0.56 \text{ cm}$$

**Zadatak 2.** Izračunaj duljinu treće stranice i ostale kutove trokuta ako je:

1)  $a = 21 \text{ cm}$ ,  $b = 15 \text{ cm}$ ,  $\alpha = 66^\circ$ ;

2)  $a = 3.8 \text{ cm}$ ,  $b = 5.9 \text{ cm}$ ,  $\beta = 64^\circ 10'$ ;

3)  $a = 0.88 \text{ cm}$ ,  $b = 1.25 \text{ cm}$ ,  $\beta = 87^\circ 36'$ ;



- 4)  $a = 10.5 \text{ cm}$ ,  $c = 13.5 \text{ cm}$ ,  $\alpha = 48^\circ 46'$ ;  
 5)  $b = 21 \text{ cm}$ ,  $c = 15 \text{ cm}$ ,  $\gamma = 36^\circ$ ;  
 6)  $a = 13.8 \text{ cm}$ ,  $c = 8 \text{ cm}$ ,  $\gamma = 15^\circ$ .

Rješenje.

- 1)  $a = 21 \text{ cm}$   
 $b = 15 \text{ cm}$   
 $\alpha = 66^\circ$

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$c, \beta, \gamma = ?$

$$\sin \beta = \frac{b \cdot \sin \alpha}{a} = \frac{15 \text{ cm} \cdot \sin 66^\circ}{21 \text{ cm}} = 0.65253$$

$$\beta_1 = 40^\circ 44', \beta_2 = 180^\circ - 40^\circ 44' = 139^\circ 16' \quad (\beta_2 \text{ nije rješenje jer } \alpha + \beta_2 > 180^\circ)$$

$$\beta = 40^\circ 44'$$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 66^\circ - 40^\circ 44' = 73^\circ 16'$$

$$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = \frac{21 \text{ cm} \cdot \sin 73^\circ 16'}{\sin 66^\circ} = 22.01 \text{ cm}$$

- 2)  $a = 3.8 \text{ cm}$   
 $b = 5.9 \text{ cm}$   
 $\beta = 64^\circ 10'$

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$c, \alpha, \gamma = ?$

$$\sin \alpha = \frac{a \cdot \sin \beta}{b} = \frac{3.8 \text{ cm} \cdot \sin 64^\circ 10'}{5.9 \text{ cm}} = 0.579703$$

$$\alpha_1 = 35^\circ 26', \alpha_2 = 180^\circ - 35^\circ 26' = 144^\circ 34' \quad (\alpha_2 \text{ nije rješenje jer } \alpha_2 + \beta > 180^\circ)$$

$$\alpha = 35^\circ 26'$$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 35^\circ 26' - 64^\circ 10' = 80^\circ 24'$$

$$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = \frac{3.8 \text{ cm} \cdot \sin 80^\circ 24'}{\sin 35^\circ 26'} = 6.46 \text{ cm}$$

$$a = 0.88 \text{ cm}$$

- 3)  $b = 1.25 \text{ cm}$   
 $\beta = 87^\circ 36'$

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$c, \alpha, \gamma = ?$

$$\sin \alpha = \frac{a \cdot \sin \beta}{b} = \frac{0.88 \text{ cm} \cdot \sin 87^\circ 36'}{1.25 \text{ cm}} = 0.703382$$

$$\alpha_1 = 44^\circ 42', \alpha_2 = 180^\circ - 44^\circ 42' = 135^\circ 26' \quad (\alpha_2 \text{ nije rješenje jer } \alpha_2 + \beta > 180^\circ)$$

$$\alpha = 44^\circ 42'$$

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 44^\circ 42' - 87^\circ 36' = 47^\circ 42'$$

$$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = \frac{0.88 \text{ cm} \cdot \sin 47^\circ 42'}{\sin 44^\circ 42'} = 0.93 \text{ cm}$$

$$4) \quad a = 10.5 \text{ cm}$$

$$c = 13.5 \text{ cm}$$

$$\alpha = 48^\circ 46'$$

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$$b, \beta, \gamma = ?$$

$$\sin \gamma = \frac{c \cdot \sin \alpha}{a} = \frac{13.5 \text{ cm} \cdot \sin 48^\circ 46'}{10.5 \text{ cm}} = 0.703382$$

$$\gamma = 75^\circ 13' \quad \text{ili} \quad \gamma = 180^\circ - 75^\circ 13' = 104^\circ 47'$$

za  $\gamma = 75^\circ 13'$  dobije se:

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 48^\circ 46' - 75^\circ 13' = 56^\circ 1'$$

$$b = \frac{a \sin \beta}{\sin \alpha} = \frac{10.5 \text{ cm} \cdot \sin 56^\circ 1'}{\sin 48^\circ 46'} = 11.59 \text{ cm}$$

za  $\gamma = 104^\circ 47'$  dobije se:

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 48^\circ 46' - 104^\circ 47' = 26^\circ 27'$$

$$b = \frac{a \sin \beta}{\sin \alpha} = \frac{10.5 \text{ cm} \cdot \sin 26^\circ 27'}{\sin 48^\circ 46'} = 6.22 \text{ cm}$$

$$5) \quad b = 21 \text{ cm}$$

$$c = 15 \text{ cm}$$

$$\gamma = 36^\circ$$

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$$a, \alpha, \beta = ?$$

$$\sin \beta = \frac{b \cdot \sin \gamma}{c} = \frac{21 \text{ cm} \cdot \sin 36^\circ}{15 \text{ cm}} = 0.822899$$

$$\beta = 55^\circ 22' 34'' \quad \text{ili} \quad \beta = 180^\circ - 55^\circ 22' 34'' = 124^\circ 37' 26''$$

za  $\beta = 55^\circ 22' 34''$  dobije se:

$$\alpha = 180^\circ - \beta - \gamma = 180^\circ - 55^\circ 22' 34'' - 36^\circ = 88^\circ 37' 26''$$

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{15 \text{ cm} \cdot \sin 88^\circ 37' 26''}{\sin 36^\circ} = 25.51 \text{ cm}$$

za  $\beta = 124^\circ 37' 26''$  dobije se:

$$\alpha = 180^\circ - \beta - \gamma = 180^\circ - 124^\circ 37' 26'' - 36^\circ = 19^\circ 12' 34''$$

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{15 \text{ cm} \cdot \sin 19^\circ 12' 34''}{\sin 36^\circ} = 8.47 \text{ cm}$$

$$6) \quad a = 13.8 \text{ cm}$$

$$c = 8 \text{ cm}$$

$$\gamma = 15^\circ$$

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$$b, \alpha, \beta = ?$$

$$\sin \alpha = \frac{a \cdot \sin \gamma}{c} = \frac{13.8 \text{ cm} \cdot \sin 15^\circ}{8 \text{ cm}} = 0.4464629$$

$$\alpha = 26^\circ 31' \quad \text{ili} \quad \alpha = 180^\circ - 26^\circ 31' = 153^\circ 29'$$

za  $\alpha = 26^\circ 31'$  dobije se:

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 26^\circ 31' - 15^\circ = 138^\circ 29'$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{8 \text{ cm} \cdot \sin 138^\circ 29'}{\sin 15^\circ} = 20.5 \text{ cm}$$

za  $\alpha = 153^\circ 29'$  dobije se:

$$\beta = 180^\circ - \alpha - \gamma = 180^\circ - 153^\circ 29' - 15^\circ = 11^\circ 31'$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{8 \text{ cm} \cdot \sin 11^\circ 31'}{\sin 15^\circ} = 6.17 \text{ cm}$$

**Zadatak 3.** Ako su  $a$  i  $b$  duljine stranica, a  $\alpha$  i  $\beta$  tim stranicama suprotni kutovi te ako vrijedi  $\frac{a}{\cos \alpha} = \frac{b}{\cos \beta}$ , taj je trokut jednakokračan. Dokaži.

*Rješenje.* Tvrdnja: ako za trokut vrijedi  $\frac{a}{\cos \alpha} = \frac{b}{\cos \beta}$ , taj je trokut jednakokračan.

$$\left. \begin{array}{l} \frac{a}{b} = \frac{\cos \alpha}{\cos \beta} \text{ pretpostavka zadatka} \\ \frac{a}{b} = \frac{\sin \alpha}{\sin \beta} \end{array} \right\} \Rightarrow \frac{\cos \alpha}{\cos \beta} = \frac{\sin \alpha}{\sin \beta}$$

$$\cos \alpha \sin \beta = \cos \beta \sin \alpha$$

$$\cos \alpha \sin \beta - \cos \beta \sin \alpha = 0$$

$$-\sin(\alpha - \beta) = 0$$

$$\sin(\alpha - \beta) = 0 \Rightarrow \alpha - \beta = 0 \Rightarrow \alpha = \beta.$$

**Zadatak 4.** Kutovi trokuta u omjeru su  $3 : 5 : 7$ . Koliki je omjer duljina najdulje i najkraće stranice trokuta?

*Rješenje.*

$$\alpha : \beta : \gamma = 3 : 5 : 7$$

$$\frac{c}{a} = ?$$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha = 3k, \quad \beta = 5k, \quad \gamma = 7k$$

$$3k + 5k + 7k = 180^\circ$$

$$15k = 180^\circ$$

$$k = 12^\circ$$

$$\alpha = 36^\circ, \quad \beta = 60^\circ, \quad \gamma = 84^\circ$$

$$\frac{c}{a} = \frac{\sin \gamma}{\sin \alpha} = \frac{\sin 84^\circ}{\sin 36^\circ} = 1.69$$