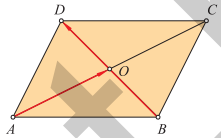
5 Vektori

5.1. Osnovni pojmovi o vektorima

Zadatak 1. Koje su od sljedećih veličina vektorske, a koje skalarne: temperatura, obujam, brzina, masa, ubrzanje, sila, električni napon?

Rješenje. Vektorske veličine su brzina, ubrzanje i sila, a skalarne temperatura, obujam, masa i električni napon.

Zadatak 2. Dan je paralelogram $ABCD$. Točka O sjecište je njegovih dijagonala. Promatramo skup vektora kojima su početna i završna točka vrh paralelograma ili točka O.



1) Ispiši sve vektore koji imaju jednak smjer kao i vektor $\overrightarrow{AO}$. Ispiši sve vektore koji imaju jednaku orijentaciju kao i vektor $\overrightarrow{AO}$.

2) Ispiši sve vektore koji imaju jednak smjer kao i vektor $\overrightarrow{BD}$. Ispiši sve vektore koji imaju jednaku orijentaciju kao i vektor $\overrightarrow{BD}$.

Rješenje. **1)** Vektori koji imaju jednak smjer kao i vektor $\overrightarrow{AO}$ su $\overrightarrow{OA}$, $\overrightarrow{AC}$, $\overrightarrow{CA}$, $\overrightarrow{OC}$ i $\overrightarrow{CO}$.

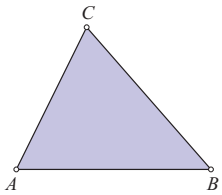
Vektori koji imaju jednaku orijentaciju kao i vektor $\overrightarrow{AO}$ su $\overrightarrow{AC}$ i $\overrightarrow{OC}$.

2) Vektori koji imaju jednak smjer kao i vektor $\overrightarrow{BD}$ su $\overrightarrow{DB}$, $\overrightarrow{BO}$, $\overrightarrow{OB}$, $\overrightarrow{DO}$ i $\overrightarrow{OD}$.

Vektori koji imaju jednaku orijentaciju kao i vektor $\overrightarrow{BD}$ su $\overrightarrow{BO}$ i $\overrightarrow{OD}$.

Zadatak 3. Koliko postoji vektora kojima su početna i završna točka neka dva vrha trokuta ABC?

Rješenje. Iz svakog vrha se mogu povući dva vektora, a s obzirom da $\overrightarrow{AB} \neq \overrightarrow{BA}$ postoji $2 \cdot 3 = 6$.



Zadatak 4. Koliko postoji vektora kojima su početna i završna točka vrhovi četverokuta $ABCD$ ako je taj četverokut paralelogram, a koliko ako nije paralelogram?

Rješenje. Iz svakog vrha se mogu povući tri vektora (u bilo kojem četverokutu), a s obzirom da $\overrightarrow{AB} \neq \overrightarrow{BA}$ postoji $3 \cdot 4 = 12$ vektora.

Zadatak 5. Ako je $\overrightarrow{AB} = \overrightarrow{CD}$, dokaži da je $\overrightarrow{AC} = \overrightarrow{BD}$.

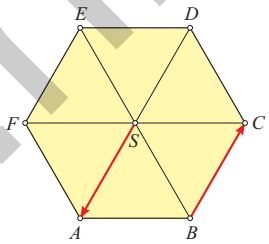
Rješenje. $\overrightarrow{AB} = \overrightarrow{CD} \implies AB \| CD$ i $|AB| = |CD|$ te su A, B, C i D vrhovi paralelograma iz čega sljedi $\overrightarrow{AC} = \overrightarrow{BD}$.

Zadatak 6. Nacrtaj pravilan šesterokut $ABCDEF$. Neka je S sjecište dijagonala tog šesterokuta. Ispiši sve vektore kojima su početna i završna točka neki vrh šesterokuta ili točka S, a koji su

1) jednaki vektoru $\overrightarrow{BC}$; **2)** suprotni vektoru $\overrightarrow{SA}$.

Rješenje. **1)** $\overrightarrow{BC} = \overrightarrow{AS}$, $\overrightarrow{SD}$, $\overrightarrow{FE}$;

2) Vektori suprotni vektoru $\overrightarrow{SA}$ su: $\overrightarrow{AS}$, $\overrightarrow{SD}$, $\overrightarrow{BC}$, $\overrightarrow{FE}$.



5.2. Zbrajanje vektora

Zadatak 1. Dan je paralelogram $ABCD$. Neka je točka S sjecište njegovih dijagonala. Izračunaj:

1) $\overrightarrow{AD} + \overrightarrow{CD}$; **2)** $\overrightarrow{AS} + \overrightarrow{BS}$; **3)** $\overrightarrow{AD} + \overrightarrow{CB}$;

4) $\overrightarrow{AB} + \overrightarrow{SD}$; **5)** $\overrightarrow{AB} + \overrightarrow{BS}$; **6)** $\overrightarrow{BS} + \overrightarrow{CS}$.

Rješenje.

1) $\overrightarrow{AD} + \overrightarrow{CD} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$;

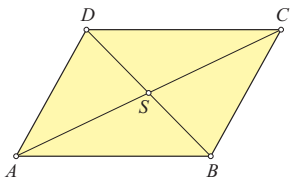
2) $\overrightarrow{AS} + \overrightarrow{BS} = \overrightarrow{AS} + \overrightarrow{SD} = \overrightarrow{AD}$;

3) $\overrightarrow{AD} + \overrightarrow{CB} = \overrightarrow{AD} + \overrightarrow{DA} = \overrightarrow{AD} - \overrightarrow{AD} = \vec{0}$;

4) $\overrightarrow{AB} + \overrightarrow{SD} = \overrightarrow{AB} + \overrightarrow{BS} = \overrightarrow{AS}$;

5) $\overrightarrow{AB} + \overrightarrow{BS} = \overrightarrow{AS}$;

6) $\overrightarrow{BS} + \overrightarrow{CS} = \overrightarrow{BS} + \overrightarrow{SA} = \overrightarrow{BA}$.

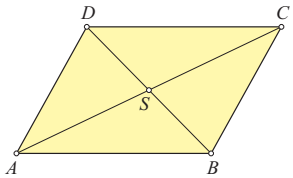


Zadatak 2. Točka S sjecište je dijagonala paralelograma $ABCD$. Izračunaj:

1) $\overrightarrow{AS} + \overrightarrow{BS} + \overrightarrow{CS}$; **2)** $\overrightarrow{AB} + \overrightarrow{CS} + \overrightarrow{BD}$;

3) $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD}$; **4)** $\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} + \overrightarrow{SD}$.

Rješenje.



1) $\overrightarrow{AS} + \overrightarrow{BS} + \overrightarrow{CS} = (\overrightarrow{AS} + \overrightarrow{SD}) + \overrightarrow{CS}$
$= \overrightarrow{AD} + \overrightarrow{CS} = \overrightarrow{BC} + \overrightarrow{CS} = \overrightarrow{BS}$;

2) $\overrightarrow{AB} + \overrightarrow{CS} + \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{SA}) + \overrightarrow{BD}$
$= (\overrightarrow{SA} + \overrightarrow{AB}) + \overrightarrow{BD} = \overrightarrow{SB} + \overrightarrow{BD} = \overrightarrow{BD} + \overrightarrow{DS} = \overrightarrow{BS}$;

3) $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = (\overrightarrow{AB} + \overrightarrow{AD}) + \overrightarrow{AC} =$ (po pravilu palalelograma) $=$
$\overrightarrow{AC} + \overrightarrow{AC} = 2\overrightarrow{AC}$;

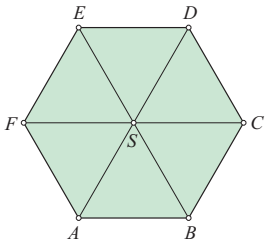
4) $\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} + \overrightarrow{SD} = (\overrightarrow{SA} + \overrightarrow{SC}) + (\overrightarrow{SB} + \overrightarrow{SD}) = (\overrightarrow{SA} - \overrightarrow{SA}) +$
$(\overrightarrow{SB} - \overrightarrow{SB}) = \vec{0}$.

Zadatak 3. Neka je $ABCDEF$ pravilni šesterokut i neka je S sjecište njegovih dijagonala. Izračunaj:

1) $\overrightarrow{AB} + \overrightarrow{EF}$; **2)** $\overrightarrow{AB} + \overrightarrow{SD}$; **3)** $\overrightarrow{BC} + \overrightarrow{ES}$;

4) $\overrightarrow{CS} + \overrightarrow{EF}$; **5)** $\overrightarrow{DE} + \overrightarrow{SC}$; **6)** $\overrightarrow{CF} + \overrightarrow{AS}$.

Rješenje.



1) $\overrightarrow{AB} + \overrightarrow{EF} = \overrightarrow{AB} + \overrightarrow{SA} = \overrightarrow{SA} + \overrightarrow{AB} = \overrightarrow{SB}$;

2) $\overrightarrow{AB} + \overrightarrow{SD} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$;

3) $\overrightarrow{BC} + \overrightarrow{ES} = \overrightarrow{ES} + \overrightarrow{BC} = \overrightarrow{ES} + \overrightarrow{SD} = \overrightarrow{ED}$;

4) $\overrightarrow{CS} + \overrightarrow{EF} = \overrightarrow{CS} + \overrightarrow{SA} = \overrightarrow{CA}$;

5) $\overrightarrow{DE} + \overrightarrow{SC} = \overrightarrow{DE} + \overrightarrow{ED} = \overrightarrow{DE} - \overrightarrow{DE} = \vec{0}$;

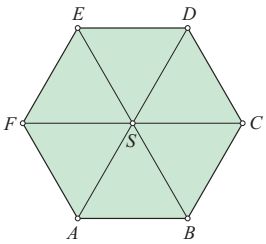
6) $\overrightarrow{CF} + \overrightarrow{AS} = \overrightarrow{CF} + \overrightarrow{FE} = \overrightarrow{CE}$.

Zadatak 4. Točka S sjecište je dijagonala pravilnog šesterokuta $ABCDEF$. Izračunaj:

1) $\overrightarrow{AB} + \overrightarrow{SD} + \overrightarrow{SF}$; **2)** $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF}$;

3) $\overrightarrow{AB} + \overrightarrow{AS} + \overrightarrow{AF}$; **4)** $\overrightarrow{SB} + \overrightarrow{SD} + \overrightarrow{SF}$.

Rješenje.



1) $\overrightarrow{AB} + \overrightarrow{SD} + \overrightarrow{SF} = (\overrightarrow{AB} + \overrightarrow{AS}) + \overrightarrow{SF}$
$=$ (po pravilu paralelograma) $= \overrightarrow{AC} + \overrightarrow{SF}$
$= \overrightarrow{AC} + \overrightarrow{CS} = \overrightarrow{AS}$;

2) $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{EF} = \overrightarrow{AB} + \overrightarrow{BS} + \overrightarrow{EF}$
$= \overrightarrow{AS} + (-\overrightarrow{AS}) = \vec{0}$;

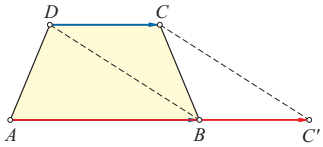
3) $\overrightarrow{AB} + \overrightarrow{AS} + \overrightarrow{AF} = (\overrightarrow{AB} + \overrightarrow{AF}) + \overrightarrow{AS} = \overrightarrow{AS} + \overrightarrow{AS} = 2\overrightarrow{AS} = \overrightarrow{AD}$;

4) $\overrightarrow{SB} + \overrightarrow{SD} + \overrightarrow{SF} = (\overrightarrow{SB} + \overrightarrow{SF}) + \overrightarrow{SD} = \overrightarrow{SA} + \overrightarrow{SD} = \overrightarrow{SA} + (-\overrightarrow{SA}) = \vec{0}$.

Zadatak 5. Dan je trapez $ABCD$. Konstruiraj vektore:

1) $\overrightarrow{AB} + \overrightarrow{DC}$; **2)** $\overrightarrow{AB} + \overrightarrow{CD}$; **3)** $\overrightarrow{AD} + \overrightarrow{CB}$; **4)** $\overrightarrow{AD} + \overrightarrow{BC}$.

Rješenje. **1)**



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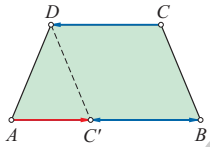

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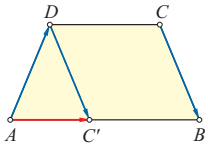

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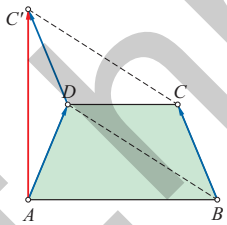


Zadatak 6. Odredi zbroj vektora:

1) $\overrightarrow{AC} + \overrightarrow{DB} + \overrightarrow{CD} + \overrightarrow{BA}$; **2)** $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{BC} + \overrightarrow{DE}$;

3) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$; **4)** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{DA}$;

5) $\overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{BC} + \overrightarrow{DB}$.

Rješenje. **1)** $\overrightarrow{AC} + \overrightarrow{DB} + \overrightarrow{CD} + \overrightarrow{BA} = (\overrightarrow{AC} + \overrightarrow{CD}) + \overrightarrow{DB} + \overrightarrow{BA}$
$= \overrightarrow{AD} + \overrightarrow{DB} + \overrightarrow{BA} = \overrightarrow{AB} + \overrightarrow{BA} = \vec{0}$;

2) $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{BC} + \overrightarrow{DE} = (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{CD} + \overrightarrow{DE}) = \overrightarrow{AC} + \overrightarrow{CE} = \overrightarrow{AE}$;

3) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA} = \vec{0}$;

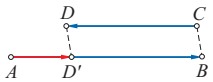
4) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{DA} = \overrightarrow{AC} + \overrightarrow{DA} = \overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$;

5) $\overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{BC} + \overrightarrow{DB} = \overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{DB} + \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

Zadatak 7. Može li zbroj vektora biti vektor manje duljine nego što je duljina svakog pojedinog pribrojnika? Može li razlika vektora biti manje duljine od njihova zbroja?

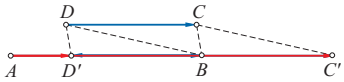
Rješenje. Zbroj vektora može biti vektor manje duljine nego što je duljina svakog pojedinog pribrojnika:

$$\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{AD'}$$



Razlika vektora može biti manje duljine od njihova zbroja:

$$\overrightarrow{AB} - \overrightarrow{DC} = \overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{AD'}$$
$$\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC'}$$



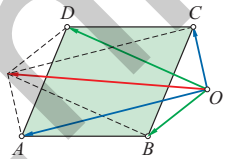
Zadatak 8. Ako točke O, A i B nisu kolinearne i ako je $\overrightarrow{OC} = \overrightarrow{OA} - \overrightarrow{OB}$, onda je $\overrightarrow{OC} = -\overrightarrow{AB}$. Dokaži.

Rješenje. $\overrightarrow{OC} = \overrightarrow{OA} - \overrightarrow{OB} = -\overrightarrow{AO} - \overrightarrow{OB} = -(\overrightarrow{AO} + \overrightarrow{OB}) = -\overrightarrow{AB}$.

Zadatak 9. Dan je paralelogram $ABCD$ i točka O. Dokaži da je $\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{OD}$.

Rješenje.

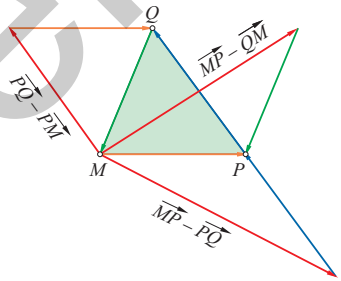
$$\begin{aligned}\overrightarrow{OA} + \overrightarrow{OC} &= \overrightarrow{OB} + \overrightarrow{BA} + \overrightarrow{OD} + \overrightarrow{DC}\\ &= \overrightarrow{OB} + \overrightarrow{OD} + \overrightarrow{BA} - \overrightarrow{BA}\\ &= \overrightarrow{OB} + \overrightarrow{OD}.\end{aligned}$$



Zadatak 10. Dan je trokut MPQ. Konstruiraj vektore:

1) $\overrightarrow{MP} - \overrightarrow{PQ}$; **2)** $\overrightarrow{MP} - \overrightarrow{QM}$; **3)** $\overrightarrow{PQ} - \overrightarrow{MP}$.

Rješenje.



Zadatak 11. Nacrtaj paralelogram $ABCD$ i odredi njegovo središte S. Izračunaj:

1) $\overrightarrow{BC} - \overrightarrow{DC}$; **2)** $\overrightarrow{AB} - \overrightarrow{BC}$; **3)** $\overrightarrow{AS} - \overrightarrow{BS}$;

4) $\overrightarrow{BS} - \overrightarrow{SD}$; **5)** $\overrightarrow{AC} - \overrightarrow{SC}$; **6)** $\overrightarrow{AS} - \overrightarrow{SD}$.

Rješenje.

1) $\overrightarrow{BC} - \overrightarrow{DC} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$;

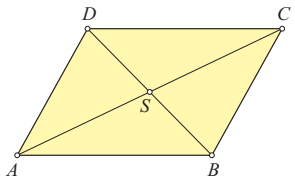
2) $\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{DA} = \overrightarrow{DA} + \overrightarrow{AB} = \overrightarrow{DB}$;

3) $\overrightarrow{AS} - \overrightarrow{BS} = \overrightarrow{AS} + \overrightarrow{SB} = \overrightarrow{AB}$;

4) $\overrightarrow{BS} - \overrightarrow{SD} = \overrightarrow{BS} - \overrightarrow{BS} = \vec{0}$;

5) $\overrightarrow{AC} - \overrightarrow{SC} = \overrightarrow{AC} + \overrightarrow{CS} = \overrightarrow{AS}$;

6) $\overrightarrow{AS} - \overrightarrow{SD} = \overrightarrow{AS} - \overrightarrow{BS} = \overrightarrow{AS} + \overrightarrow{SB} = \overrightarrow{AB}$.



Zadatak 12. Neka su A, B, C, D, E, F vrhovi pravilnog šesterokuta. Provjeri jednakosti:

1) $\overrightarrow{AB} - \overrightarrow{DC} = \overrightarrow{BC}$; **2)** $\overrightarrow{BC} - \overrightarrow{ED} = \overrightarrow{AF}$;

3) $\overrightarrow{CD} - \overrightarrow{FE} = \overrightarrow{BA}$; **4)** $\overrightarrow{AF} - \overrightarrow{DE} = \overrightarrow{BC}$.

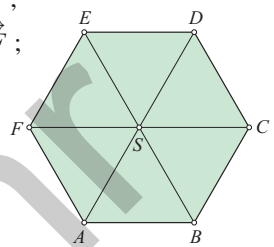
Rješenje.

1) $\overrightarrow{AB} - \overrightarrow{DC} = \overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{BS} = \overrightarrow{AS} = \overrightarrow{BC}$;

2) $\overrightarrow{BC} - \overrightarrow{ED} = \overrightarrow{BC} + \overrightarrow{DE} = \overrightarrow{BC} + \overrightarrow{CS} = \overrightarrow{BS} = \overrightarrow{AF}$;

3) $\overrightarrow{CD} - \overrightarrow{FE} = \overrightarrow{CD} + \overrightarrow{EF} = \overrightarrow{CD} + \overrightarrow{DS}$
$= \overrightarrow{CS} = \overrightarrow{BA}$;

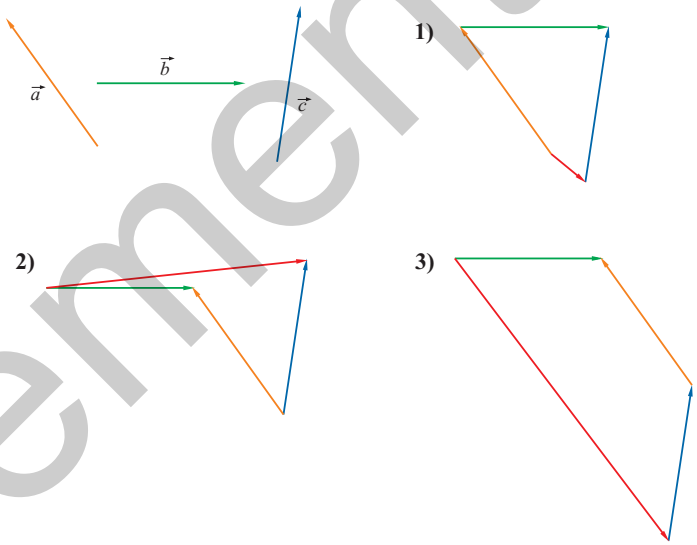
4) $\overrightarrow{AF} - \overrightarrow{DE} = \overrightarrow{AF} + \overrightarrow{ED} = \overrightarrow{AF} + \overrightarrow{FS}$
$= \overrightarrow{AS} = \overrightarrow{BC}$.



Zadatak 13. Nacrtaj neka tri vektora $\vec{a}$, $\vec{b}$ i $\vec{c}$ te konstruiraj sljedeće vektore:

1) $\vec{a} + \vec{b} - \vec{c}$; **2)** $\vec{a} - \vec{b} + \vec{c}$; **3)** $\vec{a} - \vec{b} - \vec{c}$.

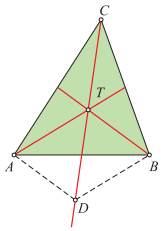
Rješenje.



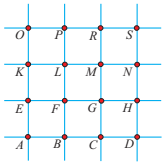
Zadatak 14. Točka T težište je trokuta ABC. Odredi zbroj vektora $\overrightarrow{TA} + \overrightarrow{TB} + \overrightarrow{TC}$.

Rješenje. Konstruiramo paralelogram $ADBT$. Tada je:

$$\overrightarrow{TA} + \overrightarrow{TB} + \overrightarrow{TC} = (\overrightarrow{TA} + \overrightarrow{TB}) + \overrightarrow{TC}$$
$$= \overrightarrow{TD} + \overrightarrow{TC} = \vec{0}.$$



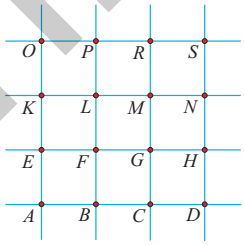
Zadatak 15. Dana je pravokutna mreža kao na slici. Odredi:



1) $\overrightarrow{AB} + \overrightarrow{KP}$; **2)** $\overrightarrow{GH} + \overrightarrow{MR}$; **3)** $\overrightarrow{AH} + \overrightarrow{SP}$;

4) $\overrightarrow{EG} + \overrightarrow{DN}$; **5)** $\overrightarrow{CF} + \overrightarrow{RN}$; **6)** $\overrightarrow{LS} + \overrightarrow{NC}$.

Rješenje.



1) $\overrightarrow{AB} + \overrightarrow{KP} = \overrightarrow{AB} + \overrightarrow{AF} = \overrightarrow{AG}$;

2) $\overrightarrow{GH} + \overrightarrow{MR} = \overrightarrow{GH} + \overrightarrow{HN} = \overrightarrow{GN}$;

3) $\overrightarrow{AH} + \overrightarrow{SP} = \overrightarrow{AH} + \overrightarrow{HF} = \overrightarrow{AF}$;

4) $\overrightarrow{EG} + \overrightarrow{DN} = \overrightarrow{EG} + \overrightarrow{GR} = \overrightarrow{ER}$;

5) $\overrightarrow{CF} + \overrightarrow{RN} = \overrightarrow{CF} + \overrightarrow{FC} = \vec{0}$;

6) $\overrightarrow{LS} + \overrightarrow{NC} = \overrightarrow{LS} + \overrightarrow{SG} = \overrightarrow{LG}$.

Zadatak 16. Promatraj pravokutnu mrežu iz prethodnog zadatka i izračunaj:

1) $\overrightarrow{EH} - \overrightarrow{KB}$; **2)** $\overrightarrow{AC} - \overrightarrow{EF}$; **3)** $\overrightarrow{NR} - \overrightarrow{MO}$;

4) $\overrightarrow{OF} - \overrightarrow{GD}$; **5)** $\overrightarrow{RO} - \overrightarrow{NC}$; **6)** $\overrightarrow{AH} - \overrightarrow{MR}$.

Rješenje.

1) $\overrightarrow{EH} - \overrightarrow{KB} = \overrightarrow{EH} + \overrightarrow{BK} = \overrightarrow{EH} + \overrightarrow{HP} = \overrightarrow{EP}$;

2) $\overrightarrow{AC} - \overrightarrow{EF} = \overrightarrow{AC} + \overrightarrow{FE} = \overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$;

3) $\overrightarrow{NR} - \overrightarrow{MO} = \overrightarrow{NR} + \overrightarrow{OM} = \overrightarrow{LO} + \overrightarrow{OM} = \overrightarrow{LM}$;

4) $\overrightarrow{OF} - \overrightarrow{GD} = \overrightarrow{OF} + \overrightarrow{DG} = \overrightarrow{OF} + \overrightarrow{FK} = \overrightarrow{OK}$;

5) $\overrightarrow{RO} - \overrightarrow{NC} = \overrightarrow{RO} + \overrightarrow{CN} = \overrightarrow{CN} + \overrightarrow{NL} = \overrightarrow{CL}$;

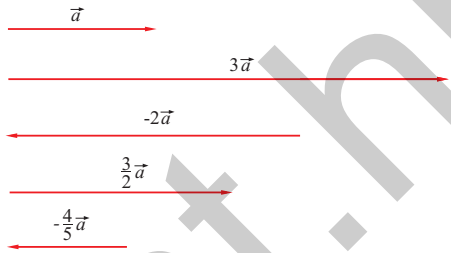
6) $\overrightarrow{AH} - \overrightarrow{MR} = \overrightarrow{AH} + \overrightarrow{RM} = \overrightarrow{AH} + \overrightarrow{HD} = \overrightarrow{AD}$.

5.3. Množenje vektora skalarom

Zadatak 1. Nacrtaj neki vektor $\vec{a}$ pa konstruiraj vektore

$$3\vec{a}, \quad -2\vec{a}, \quad \frac{3}{2}\vec{a}, \quad -\frac{4}{5}\vec{a}.$$

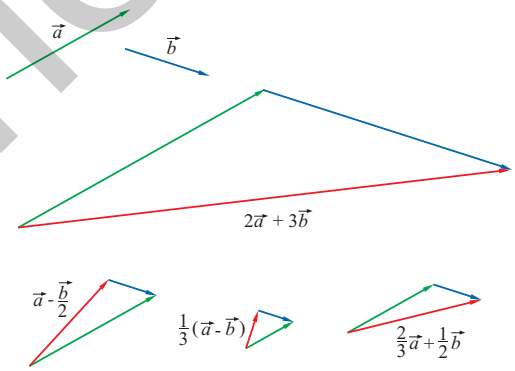
Rješenje.



Zadatak 2. Nacrtaj dva vektora $\vec{a}$ i $\vec{b}$ pa konstruiraj vektore

$$2\vec{a}+3\vec{b}, \quad \vec{a}-\frac{\vec{b}}{2}, \quad \frac{1}{3}(\vec{a}-\vec{b}), \quad \frac{2}{3}\vec{a}+\frac{1}{2}\vec{b}.$$

Rješenje.



Zadatak 3. Za koje vrijednosti broja k su vektori $\vec{a}$ i $k\cdot\vec{a}$

1) jednaki; **2)** suprotni; **3)** iste orijentacije?

Rješenje. **1)** $k=1$; **2)** $k=-1$; **3)** $k>0$.

Zadatak 4. Dan je četverokut $ABCD$. Kakav je to četverokut ako su vektori $\overrightarrow{AB}$ i $\overrightarrow{DC}$

1) jednaki; **2)** kolinearni?

Rješenje. **1)** Četverokut je paralelogram; **2)** četverokut je trapez.

Zadatak 5. Pojednostavni:

1) $\overrightarrow{AB} - \overrightarrow{BC} - \overrightarrow{CD} - \overrightarrow{DA}$;

2) $(\overrightarrow{AB} - \overrightarrow{BC}) - (\overrightarrow{CD} + \overrightarrow{AD}) + (\overrightarrow{CB} - \overrightarrow{CD})$.

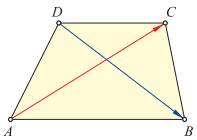
Rješenje. **1)** $\overrightarrow{AB} - \overrightarrow{BC} - \overrightarrow{CD} - \overrightarrow{DA} = \overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{DC} + \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AD} + \overrightarrow{DC}$
$= \overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB} + \overrightarrow{AB} = 2\overrightarrow{AB}$;

2) $(\overrightarrow{AB} - \overrightarrow{BC}) - (\overrightarrow{CD} + \overrightarrow{AD}) + (\overrightarrow{CB} - \overrightarrow{CD})$
$= \overrightarrow{AB} - \overrightarrow{BC} - \overrightarrow{CD} - \overrightarrow{AD} + \overrightarrow{CB} - \overrightarrow{CD}$
$= \overrightarrow{AB} + \overrightarrow{CB} + \overrightarrow{DC} + \overrightarrow{DA} + \overrightarrow{CB} + \overrightarrow{DC}$
$= \overrightarrow{DA} + \overrightarrow{AB} + 2(\overrightarrow{DC} + \overrightarrow{CB}) = \overrightarrow{DB} + 2\overrightarrow{DB} = 3\overrightarrow{DB}$.

Zadatak 6. Dan je trapez $ABCD$. Dokaži da je vektor $\overrightarrow{AC} + \overrightarrow{DB}$ kolinearan s vektorom $\overrightarrow{AB}$.

Uputa: Konstruiraj vektor jednak vektoru $\overrightarrow{DB}$ s početkom u točki C.

Rješenje. Zapišimo:
$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AD} + k \cdot \overrightarrow{AB}$,
$\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AD}$.
Tada je $\overrightarrow{AC} + \overrightarrow{DB} = (k+1)\overrightarrow{AB}$.



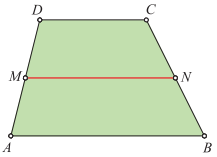
Zadatak 7. Srednjica trapeza dužina je koja spaja polovišta njegovih krakova. Srednjica je paralelna osnovicama i njezina je duljina $s = \frac{1}{2}(a+c)$, gdje su a i c duljine osnovica trapeza. Dokaži.

Rješenje. Neka su M i N polovišta krakova $\overline{AD}$, odnosno $\overline{BC}$, trapeza $ABCD$. Možemo zapisati:

$$\overrightarrow{AM} + \overrightarrow{MN} + \overrightarrow{NB} = \overrightarrow{AB},$$

te

$$\overrightarrow{DM} + \overrightarrow{MN} + \overrightarrow{NC} = \overrightarrow{DC}.$$



Nakon zbrajanja ovih dviju jednakosti dobijemo

$$2\overrightarrow{MN} = \overrightarrow{AB} + \overrightarrow{DC},$$

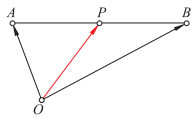
a odatle

$$\overrightarrow{MN} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{DC}).$$

Kako su vektori $\overrightarrow{AB}$ i $\overrightarrow{DC}$ kolinearni, iz ove jednakosti izravno slijedi tvrdnja iskazana u zadatku.

Zadatak 8. Ako je P polovište dužine $\overline{AB}$, a O neka točka u ravnini, tada je $\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$. Dokaži.

Rješenje. Zbrojimo li jednakosti $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$ i $\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$, dobit ćemo



$$2\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{AP} + \overrightarrow{BP}$$
$$2\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{AP} - \overrightarrow{AP}$$
$$2\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$$

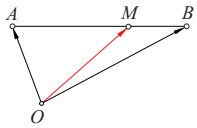
a odatle

$$\overrightarrow{OP} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}).$$

Zadatak 9. Točka M dijeli dužinu $\overline{AB}$ na dva dijela tako da je $|AM| : |MB| = 2 : 1$. Ako je O bilo koja točka, dokaži da vrijedi: $\overrightarrow{OM} = \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OB}$.

Uputa: Vidi rješenje prethodnog zadatka.

Rješenje.



$$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$
$$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM}$$
$$\left.\begin{aligned}\overrightarrow{OM} &= \overrightarrow{OA} + \overrightarrow{AM}\\ \overrightarrow{OM} &= \overrightarrow{OB} - \frac{1}{2}\overrightarrow{AM}\end{aligned}\right\} +$$
$$2\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{OB} + \frac{1}{2}\overrightarrow{AM}$$
$$2\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{OB} + \frac{1}{2}(\overrightarrow{AO} + \overrightarrow{OM})$$
$$2\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{OB} - \frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OM}$$
$$\frac{3}{2}\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OA} + \overrightarrow{OB} \quad / \cdot \frac{2}{3}$$
$$\overrightarrow{OM} = \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OB}$$

Zadatak 10. Neka je $ABCD$ paralelogram, točka O sjecište dijagonala paralelograma i T bilo koja točka u ravnini. Tada vrijedi $\overrightarrow{TO} = \frac{1}{4}(\overrightarrow{TA} + \overrightarrow{TB} + \overrightarrow{TC} + \overrightarrow{TD})$. Dokaži.

Rješenje.

$$\overrightarrow{TO} = \overrightarrow{TA} + \overrightarrow{AO} = \overrightarrow{TA} + \frac{1}{2}\overrightarrow{AC}$$
$$= \overrightarrow{TA} + \frac{1}{2}(\overrightarrow{AT} + \overrightarrow{TC})$$
$$= \overrightarrow{TA} - \frac{1}{2}\overrightarrow{TA} + \frac{1}{2}\overrightarrow{TC} = \frac{1}{2}(\overrightarrow{TA} + \overrightarrow{TC})$$